

KEMENTERIAN PENGAJIAN TINGGI JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

MATHEMATICAL COMPUTING: NUMBERING SYSTEM

FOR MALAYSIAN POLYTECHNIC STUDENTS

NORMI ISMAIL | MELATI SABTU DEPARTMENT OF MATHEMATICS, SCIENCE AND COMPUTER POLITEKNIK KUALA TERENGGANU, TERENGGANU

BY:

KUALA TERENGGANU

This page intentionally left blank.

MATHEMATICAL COMPUTING: NUMBERING SYSTEM

FOR MALAYSIAN POLYTECHNIC STUDENTS

By:

Normi Ismail | Melati Sabtu Department of Mathematics, Science and Computer Politeknik Kuala Terengganu, Terengganu Copyright © 2022 Politeknik Kuala Terengganu www.pkt.edu.my

Normi Ismail | Melati Sabtu Department of Mathematics, Science and Computer Politeknik Kuala Terengganu, Terengganu

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, photocopying, recording or otherwise, without the prior written permission of the Kuala Terengganu Polytechnic and Department of Polytechnic Education and Community Colleges, Ministry of Higher Education Malaysia.

Published by: Politeknik Kuala Terengganu Polytechnic Jalan Sultan Ismail, 20200 Kuala Terengganu, Terengganu

e-ISBN 978-967-2240-33-4

Perpustakaan Negara Malaysia	Cataloguing-in-Publication Data
Normi Ismail, 1979-	
MATHEMATICAL COMPUTING: NUN	IBERING SYSTEM FOR MALAYSIAN
POLYTECHNIC STUDENTS / By: No	rmi Ismail, Melati Sabtu.
Mode of Access: Internet	
eISBN 978-967-2240-33-4	
1. Mathematics.	
2. Numeration.	
3. Decimal system.	
4. Computer scienceMathematics.	
5. Government publicationsMalays	ia.
6. Electronic books.	
I. Melati Sabtu, 1980-	
II. Title.	
510	



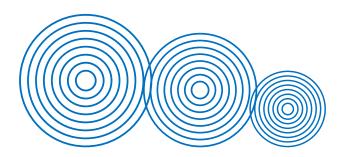
ACKNOWLEDGEMENT

السلام عليكم

With the name of Allah, Most Gracious, Most Merciful, the First and the Foremost. Our deepest gratitude extends to Allah S.W.T who has given us patience, strength, determination and courage to carry out the writing of this **MATHEMATICAL COMPUTING: NUMBERING SYSTEM FOR MALAYSIAN POLYTECHNIC STUDENTS** e-book.

This e-book is a collaborative effort of many parties. Many thanks and appreciation are extended to all the partners of the Department of Mathematics, Science and Computer, Politeknik Kuala Terengganu for their views, helpful cooperation and encouraging comments. Finally, we are very proud and hope that this e-book can benefit the community, especially students and lecturers.

Thank you.



ABSTRACT



MATHEMATICAL COMPUTING: NUMBERING SYSTEM FOR MALAYSIAN POLYTECHNIC STUDENTS

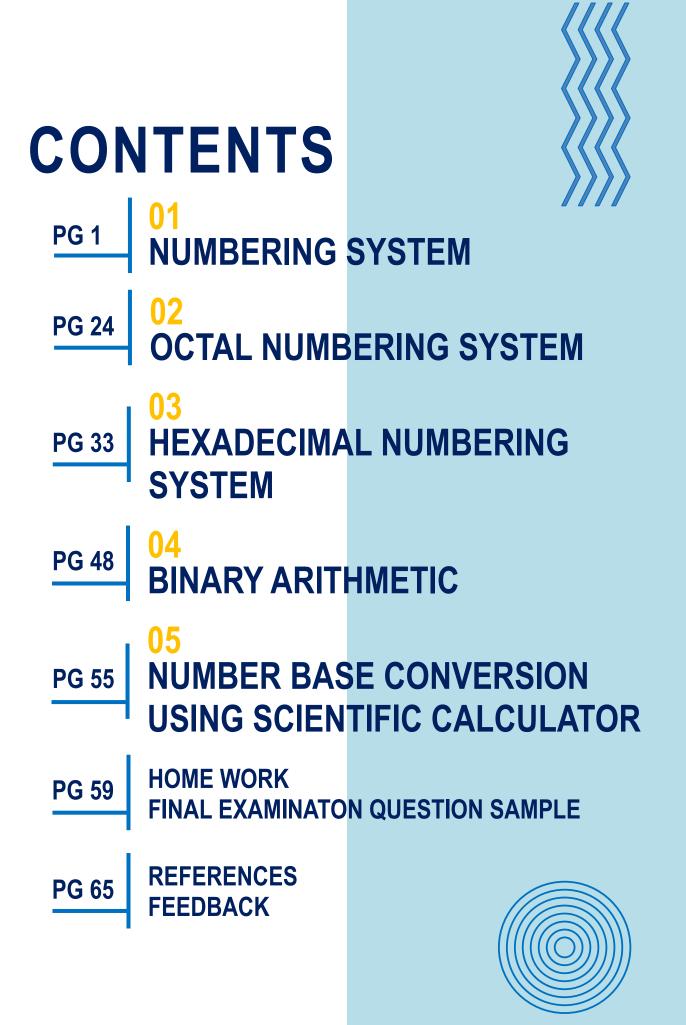
This book introduces students to numbering system, octal numbering system, hexadecimal numbering system and binary arithmetic. This book emphasizes the binary, octal and hexadecimal numbering systems used in computer science and computer programming. It introduces the way numbers are represented in each of these systems, how to switch between them (and to and from base 10).

Therefore, this book contains FIVE (5) main chapters, various examples of work and tutorials to enhance students learning abilities. The authors hope this book can provide useful resources to students and lecturers.

This book is the full property of Politeknik Kuala Terengganu which is used on the online / offline learning platform. The production of this book is also suitable for use by community college, pre-diploma and university students.

Any question can contact us at the email address:

- normi.pkt@celt.edu.my
- melatisabtu@gmail.com





01

NUMBERING SYSTEM

1.1

Decimal, Binary, Octal and Hexadecimal

1.2

Different Types of Number System

1.3

Binary System

1.4

Add and Subtract in Octal and Hexadecimal 1.5

Binary to Decimal and Decimal to Binary

, 1.6

Data Organization

Bits, Nibbles, Bytes, Words and Double Words

1.7



Unit, Number, Base/Radix, Positional Notation and Most & Least Significant Digits

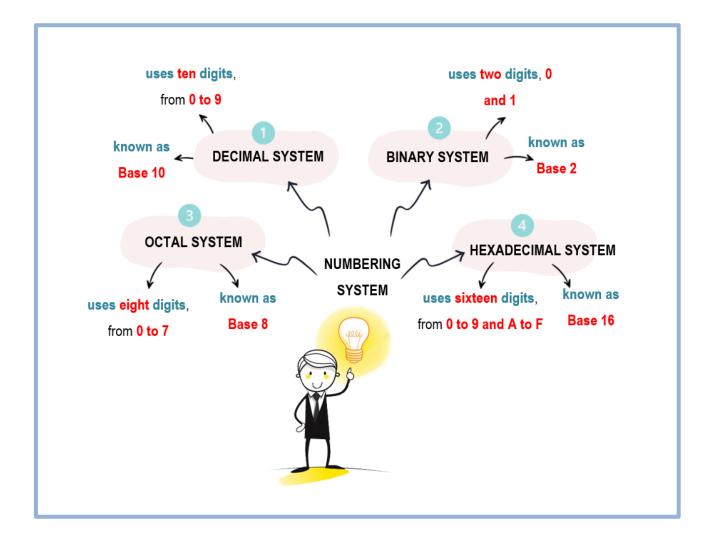
NUMBERING SYSTEM

Numbering system is a way or system representing numbers by using digits or other symbols in a consistent manner. The numbers allow us to operate arithmetic operations such as addition, subtraction, and multiplication.

1.1 DECIMAL, BINARY, OCTAL AND HEXADECIMAL

There are **four (4) main different types of number systems** which are decimal, binary, octal and hexadecimal.

- **Decimal System** | A numbering system that <u>uses ten (10) digits</u>, from 0 to 9, arranged in a series of columns to represent all numerical quantities. Each column or place value has a weighted value of 1, 10, 100, 1000, and so on, ranging from right to left. Since this system uses ten (10) digits, it has the **Base 10**.
- **Binary System** | A numbering system that <u>uses two (2) digits</u>, 0 and 1, arranged in a series of columns to represent all numerical quantities. Each column or place value has a weighted value of 1, 2, 4, 8, 16, and so on, ranging from right to left. Since this system uses two (2) digits, it has the **Base 2**.
- **Octal System** | A numbering system that <u>uses eight (8) digits</u>, 0 to 7, arranged in a series of columns to represent all numerical quantities. Each column or place value has a weighted value of 1, 8, 64, 512, and so on, ranging from right to left. Since this system uses eight (8) digits, it has the **Base 8**.
- Hexadecimal System | A numbering system that uses sixteen (16) digits, 0 to 9 and A to F, arranged in a series of columns to represent all numerical quantities. Each column or place value has a weighted value of 1, 16, 256, 4096, and so on, ranging from right to left. Since this system has sixteen (16) alphanumeric values, it has the Base 16.



MIND MAP: NUMBERING SYSTEM



SYSTEM	BASE	ALLOWED DIGITS	USED BY	
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	Human	
Binary	2	0, 1	Computer	
Octal	8	0, 1, 2, 3, 4, 5, 6, 7	Computer	
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	Computer	

1.2 DIFFERENT TYPES OF NUMBER SYSTEMS



Quantities / Counting

Dec	Hex	Oct	Bin	Dec	Hex	Oct	Bin	Dec	Hex	Oct	Bin	Dec	Hex	Oct	Bin
0	0	000	00000000	16	10	020	00010000	32	20	040	00100000	48	30	060	0011000
1	1	001	00000001	17	11	021	00010001	33	21	041	00100001	49	31	061	0011000
2	2	002	00000010	18	12	022	00010010	34	22	042	00100010	50	32	062	0011001
3	3	003	00000011	19	13	023	00010011	35	23	043	00100011	51	33	063	0011001
4	4	004	00000100	20	14	024	00010100	36	24	044	00100100	52	34	064	0011010
5	5	005	00000101	21	15	025	00010101	37	25	045	00100101	53	35	065	0011010
5 6	6		00000110	22	16		00010110	38	26		00100110	54	36		0011011
7	7		00000111	23	17		00010111	39	27		00100111	55	37		0011011
8	8	010	00001000	24	18	030	00011000	40	28	050	00101000	56	38	070	0011100
9	9	011	00001001	25	19	031	00011001	41	29	051	00101001	57	39	071	0011100
10	A	012	00001010	26	1A	032	00011010	42	2A	052	00101010	58	3A	072	0011101
11	В		00001011	27	1B		00011011	43	2B		00101011	59	3B		0011101
12	C	014	00001100	28	1C	034	00011100	44	2C	054	00101100	60	3C	074	0011110
13	D	015	00001101	29	1D		00011101	45	2D		00101101	61	3D		0011110
14	E	016	00001110	30	1E	036	00011110	46	2E	056	00101110	62	3E	076	0011111
15	F	017	00001111	31	1F	037	00011111	47	2F	057	00101111	63	3F	077	0011111
Dec	Hex	Oct	Bin	Dec	Hex	Oct	Bin	Dec	Hex	Oct	Bin	Dec	Hex	Oct	Bin
64	40	100	01000000	80	50	120	01010000	96	60	140	01100000	112	70	160	0111000
65	41	101	01000001	81	51	121	01010001	97	61	141	01100001	113	71	161	0111000
66	42	102	01000010	82	52	122	01010010	98	62	142	01100010	114	72	162	0111001
67	43	103	01000011	83	53	123	01010011	99	63	143	01100011	115	73	163	0111001
68	44	104	01000100	84	54	124	01010100	100	64	144	01100100	116	74	164	0111010
69	45	105	01000101	85	55	125	01010101	101	65	145	01100101	117	75	165	0111010
70	46	106	01000110	86	56	126	01010110	102	66	146	01100110	118	76	166	0111011
71	47	107	01000111	87	57	127	01010111	103	67	147	01100111	119	77	167	0111011
72	48	110	01001000	88	58	130	01011000	104	68	150	01101000	120	78	170	0111100
73	49	111	01001001	89	59	131	01011001	105	69	151	01101001	121	79	171	0111100
74	4A	112	01001010	90	5A	132	01011010	106	6A	152	01101010	122	7A	172	0111101
75	4B	113	01001011	91	5B	133	01011011	107	6B	153	01101011	123	7B	173	0111101
76	4C	114	01001100	92	5C	134	01011100	108	6C	154	01101100	124	7C	174	0111110
77	4D	115	01001101	93	5D	135	01011101	109	6D	155	01101101	125	7D	175	0111110
	4E	116	01001110	94	5E	136	01011110	110	6E	156	01101110	126	7E	176	0111111
78															

1.3 **BINARY SYSTEM**

A binary system is made up Binary Digits. A Binary Digit can only be 0 and 1, arranged in a series of columns to represent all numerical quantities.

Binary numbers have many uses in mathematics and beyond. In the computer world "**binary digit**" is abbreviated to the word "**bit**".



Each bit in the binary system has a <u>place value</u> and a <u>weighted value</u> of 1, 2, 4, 8, 16, and so on assigned to it, ranging from right to left.



Binary System Application

Binary Systems are very useful in computer technology and computer programming languages that help in digital encoding. The Binary System can also be used in Boolean algebra.

Advantages and disadvantages

- The main advantage of using binary is that it is a base which is easily represented by electronic devices. The Binary System also simplifies use in coding, fewer computations and less computational errors.
- The main disadvantage of using binary is difficult to read and write for humans because of large number of binary with an equivalent decimal numbers.

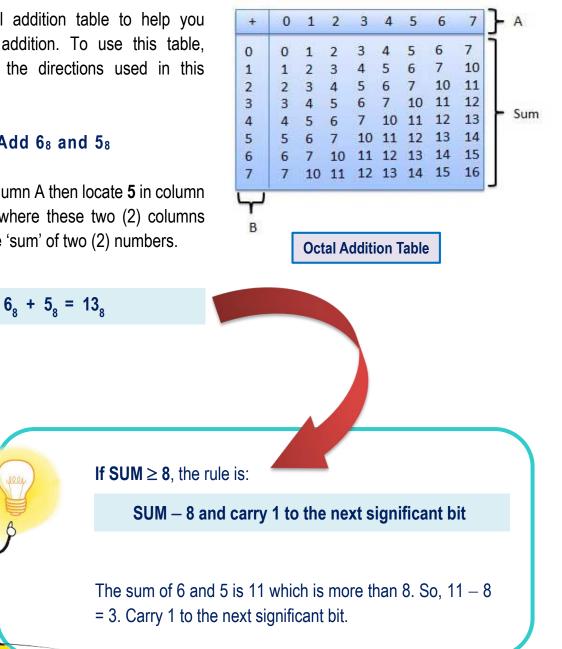
1.4 ADD AND SUBTRACT IN OCTAL AND HEXADECIMAL

OCTAL ADDITION

Use the octal addition table to help you handle octal addition. To use this table, simply follow the directions used in this example:

Add 68 and 58

Locate 6 in column A then locate 5 in column B. The point where these two (2) columns intersect is the 'sum' of two (2) numbers.





Example 1

Calculate 1628 + 5378 =?

Solution:

∴ 162₈ + 537₈ = 721₈

Example 2

Calculate 1368 + 6368 =?

Solution:

∴ 136₈ + 636₈ = 774₈



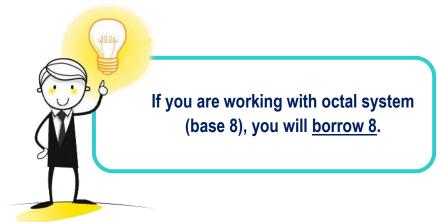
MATH DRILLS!

Calculate the followings without using calculator.

a. 456 ₈ + 123 ₈	b. 423 ₈ + 326 ₈	c. 354 ₈ + 506 ₈
Answer: 601 ₈	Answer: 7518	Answer: 1062 ₈
d. 134 ₈ + 427 ₈	e. 326 ₈ + 623 ₈	f. 656 ₈ + 177 ₈
Answer: 5638	Answer: 11518	Answer: 10558
g. 6712 ₈ + 15 ₈	h. 1023 ₈ + 424 ₈	i. 1456 ₈ + 5112 ₈
Answer: 6727 ₈	Answer: 1447 ₈	Answer: 5112 ₈
j. 56712 ₈ + 215 ₈	k. 11111₃ + 65302₃	I. 26662 ₈ + 75442 ₈
Answer: 57127 ₈	Answer: 764138	Answer: 124324 ₈

OCTAL SUBTRACTION

The subtraction of octal numbers follows the same rules as the subtraction of numbers in any other number system. The only variation is in borrowed numbers. Borrow is equal to the base of number system.





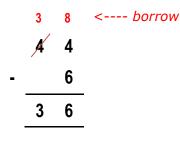
<u>Example 1</u>

Example 2

Calculate $44_8 - 6_8 = ?$

Calculate 4568 - 1738 =?

Solution:



 $\therefore 44_8 - 6_8 = 36_8$

Solution:

	3	8	< borrow
	Å	5	6
-	1	7	3
	2	6	3

∴ 456₈ - 173₈ = 263₈



MATH DRILLS!

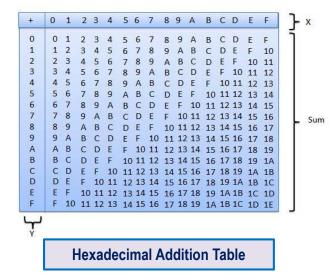
Calculate the followings without using calculator.

a 532. 171.	b. 423 ₈ - 326 ₈	c. 712 ₈ – 15 ₈
a. 532 ₈ – 174 ₈	U. 4238 - 3208	0.1128 - 108
Answer: 336 ₈	Answer: 75 ₈	Answer: 675 ₈
d. $1023_8 - 424_8$	e. 100 ₈ – 24 ₈	f. 273 ₈ – 127 ₈
Answer: 3778	Answer: 548	Answer: 144 ₈
g. 7243 ₈ – 4124 ₈	h. 335 ₈ – 224 ₈	i. 6453 ₈ − 721 ₈
Answer: 31178	Answer: 1118	Answer: 55328
j. 4343 ₈ – 526 ₈	k. 250 ₈ – 40 ₈	I. 23038 – 1168
Answer: 3615 ₈	Answer: 2108	Answer: 21658

HEXADECIMAL ADDITION

Uses 10 digits and 6 letters, **0**, **1**, **2**, **3**, **4**, **5**, **6**, **7**, **8**, **9**, **A**, **B**, **C**, **D**, **E**, **F**. Letters represents numbers starting from 10.

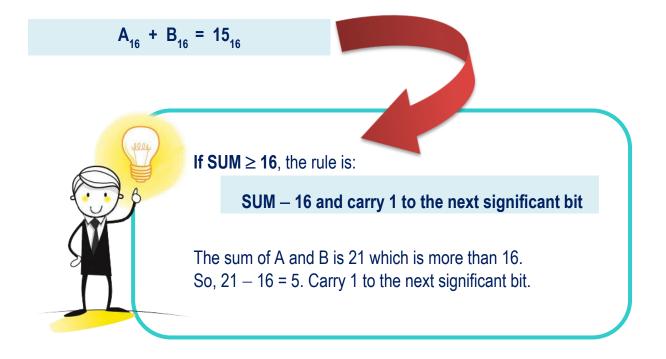
A = 10, B = 11, C = 12, D = 13, E = 14, F = 15



To use this table, simply follow the directions used in this example:

Add A₁₆ and B₁₆

Locate **A** in column X then locate **B** in column Y. The point where these two (2) columns intersect is the 'sum' of two (2) numbers.

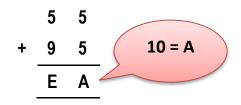




Example 1

Calculate 55₁₆ + 95₁₆ =?

Solution:



 $\therefore 55_{16} + 95_{16} = EA_{16}$

Example 2

Calculate 4A6₁₆ + 1B3₁₆ =?

Solution:

	1		< cai	тy
	4	Α	6	
+	1	В	3	
	6	5	9	

 $\therefore 4A6_{16} + 1B3_{16} = 659_{16}$



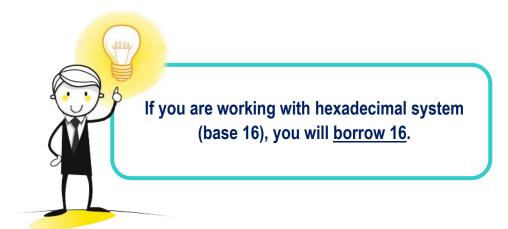
MATH DRILLS!

Calculate the followings without using calculator.

a. 4A ₁₆ + 1F ₁₆	b. B23 ₁₆ + 326 ₁₆	C. 354 ₁₆ + 5E6 ₁₆
Answer: 6916 d. 13416 + 42716	Answer: E49 ₁₆ e. 326 ₁₆ + 623 ₁₆	Answer: 93A16 f. 65616 + 17716
Answer: 55B ₁₆ g. 6712 ₁₆ + 15 ₁₆	Answer: 949 ₁₆ h. 1023 16 + 424 16	Answer: 7CD ₁₆ i. 1456 ₁₆ + 5112 ₁₆
Answer: 672716	Answer: 1447 ₁₆	Answer: 656816
Answer: 6727 ₁₆ j. ABC ₁₆ + DEF ₁₆	k. FEB ₁₆ + 123 ₁₆	I. 9A ₁₆ + 9E ₁₆
Answer: 18AB ₁₆	Answer: 110E ₁₆	Answer: 138 ₁₆

HEXADECIMAL SUBTRACTION

The subtraction of hexadecimal numbers follows the same rules as the subtraction of numbers in any other number system. The only variation is in borrowed numbers. Borrow is equal to the base of number system.





Example 1

Example 2

Calculate 5CD2₁₆ - 2A0₁₆ =?

Calculate 4A6₁₆ - 1B3₁₆ =?

Solution:

5	С	D	2
	2	Α	0
5	Α	3	2

∴ 5CD2₁₆ - 2A0₁₆ = 5A32₁₆

Solution:

	3	16	< borrow
	A	Α	6
•	1	В	3
	2	F	3



MATH DRILLS!

Calculate the followings without using calculator.

a. 5F2 ₁₆ – 174 ₁₆	b. A23 ₁₆ – 26 ₁₆	c. 56B ₁₆ – 24 ₁₆
Answer: 47E ₁₆	Answer: 9FD ₁₆	Answer: 547 ₁₆
d. 1023 ₁₆ – 424 ₁₆	e. 100 ₁₆ – 24 ₁₆	f. 273 ₁₆ – 127 ₁₆ Answer: 14C ₁₆
g. 7E43 ₁₆ – 124 ₁₆	h. 3BC ₁₆ – 224 ₁₆	i. ABCD ₁₆ – EF ₁₆
Answer: 7D1F16	Answer: 19816	Answer: AADE16
j. 4343 ₁₆ – 5B6 ₁₆	k. F50 ₁₆ – 40 ₁₆	I. 2303 ₁₆ – 116 ₁₆
Answer: 3D8D16	Answer: F10 ₁₆	Answer: 21ED ₁₆

1.5 BINARY TO DECIMAL AND DECIMAL TO BINARY

BINARY TO DECIMAL

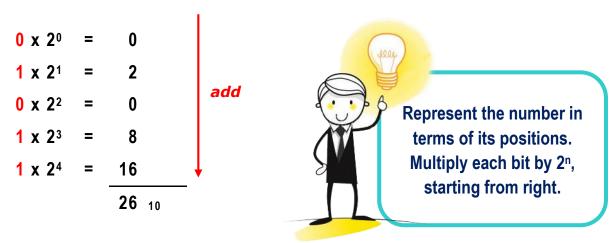
To convert binary to decimal, basic knowledge of how to read binary numbers may help. In the positional system of binary, each bit (binary digit) is a power of 2. This means that each binary number can be **represented as powers of 2**, with the rightmost being in position 2^o.



Method 1:

 $(1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$ = 16 + 8 + 0 + 2 + 0 = 26₁₀

Method 2:



∴11010₂ = 26₁₀



MATH DRILLS!

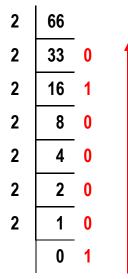
Convert the followings binary numbers into decimal.

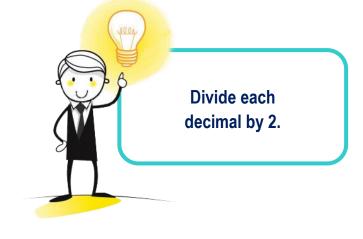
a 11001	h 10010	o 11111
a. 11001₂	b. 100102	c. 11111 ₂
Answer: 25	Answer: 18	Answer: 31
d. 110001 ₂	e. 110111 ₂	f. 101110 ₂
Answer: 49	Answer: 55	Answer: 46

DECIMAL TO BINARY

One of the methods to convert decimal to binary is **by dividing the given decimal number recursively by 2**. Then, the remainders are noted down till we get 0 as the final quotient. After this step, these remainders are written in reverse order to get the binary value of the given decimal number.







 \therefore 66₁₀ = 1000010₂



MATH DRILLS!

Convert the followings decimal numbers into binary.

a. 23 ₁₀	b. 37 ₁₀	c. 40 ₁₀
Answer: 101112	Answer: 1001012	Answer: 101000 ₂
d. 51 ₁₀	e. 64 ₁₀	f. 76 ₁₀
Answer: 110011 ₂	Answer: 10000002	Answer: 1001100 ₂

1.6 DATA ORGANIZATION

In pure mathematics a value may take an arbitrary number of bits. Computers, on the other hand, generally work with some specific number of bits.

Common collections are:



A single numerical unit in the binary number system. Bit is short for **BI**nary digi**T**. It is the smallest unit of digital data.



Four (4) bits in series, or half an 8-bit byte. A nibble is used to represent decimal numbers in BCD (binary coded decimal) or hexadecimal in binary.

3 Bytes

A series of eight (8) bits.

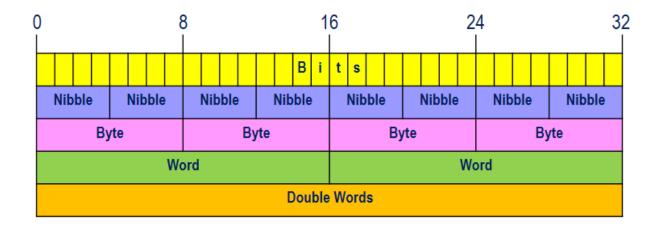


A group of sixteen (16) bits.

Double Words

A pair of words which have thirty-two (32) bits.

The following diagram shows the relative sizes of the most commonly-sized "collections" of binary information:



EXAMPLES >> HOW TO CONVERT							
Word to Nibble	Nibble to Word						
MULTIPLY the digital storage value by 4 (1 word = 4 nibbles)	DIVIDE the digital storage value by 4						
Example: Convert 11 words to nibble.	Example: Convert 36 nibbles to word.						
Solution: 11 word = 11 × 4 nibble = 44 nibble	Solution: 36 nibble = 36 / 4 word = 9 word						
Word to Bit	Bit to Word						
MULTIPLY the digital storage value by 16 (1 word = 16 bits)	DIVIDE the digital storage value by 16						
Example: Convert 7 words to bit.	Example: Convert 28 bits to word.						
Solution : 7 word = 7 × 16 bit = 112 bit	Solution : 28 nibble = 24 / 16 word = 1.5 word						





MATH DRILLS!

Write the followings in terms of data organization stated in bracket.

a. 16 bits [ni	bbles] b. 4 words	[nibbles] c.	5 bytes [nibbles]
Answer: 4	4 nibbles Answ	er: 16 nibbles	Answer: 10 nibbles
d. 10 bytes	[bits] e. 2 words	[bits] f.	23 nibbles [bits]
Answe	er: 80 bits A	nswer: 32 bits	Answer: 92 bits
g. 48 bits [word] h. 100 nibble	es [word] i.	2016 bits [bytes]
Answer	: 3 words Ans	wer: 25 words	Answer: 252 bytes

1.7 UNIT, NUMBER, BASE/RADIX, POSITIONAL NOTATION, MOST AND LEAST SIGNIFICANT DIGITS

Unit and Number

- Simplest terms, the basis upon which all other numbers are defined.
- Unit is a single object. A number is a symbol representing a unit or a quantity.

Base / Radix

- The base of a system, more properly called the RADIX (r), is the number of different values that can be expressed using a single digit including zero (0) in the number system.
- A number of system radix (r), typically has a set of r allowed digits belongs to {0, 1, ..., r 1}. Therefore, the decimal system has a radix of 10, the octal system has a radix of 8, hexadecimal is radix 16, and binary is radix 2.

Decimal	Octal	Hexadecimal	Binary
Base = 10 → r	Base = $8 \rightarrow r$	Base = 16 → r	Base = $2 \rightarrow r$
0 to r − 1	0 to r − 1	0 to r − 1	0 to r − 1
0 to 10 – 1	0 to 8 – 1	0 to 16 – 1	0 to 2 – 1
=> 0 to 9	=> 0 to 7	=> 0 to F	=> 0 to 1

Positional Notation

- A system of expressing number in which the digits are arranged in succession, the position of each digit has a place value, and the number is equal to the sum of the products of each digit by its place value.
- Example:

 $953 = (9 \times 10^{2}) + (5 \times 10^{1}) + (3 \times 10^{0}) \\ = 900 + 50 + 3 \\ = 953$

Most and Least Significant Digits

- The leftmost nonzero digit is the most significant. The rightmost digit is the least significant.
- o Examples: 1001001

1001001 ↑ ↓ ↓ Most Significant Digit Least Significant Digit



OCTAL NUMBERING SYSTEM

2.1

- Octal to Decimal
- Decimal to Octal

2.2

- Octal to Binary
- Binary to Octal



2.1 OCTAL TO DECIMAL & DECIMAL TO OCTAL

OCTAL TO DECIMAL

To convert octal to decimal, we need to <u>multiply the octal digits with the power of 8</u> starting from the rightmost being in position 8^o.



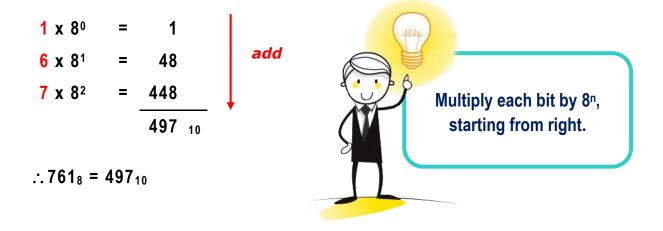
Method 1:

Represent the number in terms of its positions.

 $(7 \times 8^2) + (6 \times 8^1) + (1 \times 8^0)$ = 448 + 48 + 1 = 497₁₀

Method 2:

Represent the number in terms of its positions.





MATH DRILLS!

Convert the followings octal numbers into decimal.

a. 16₈ b. 100₈ c. 2228 Answer: 14 Answer: 64 Answer: 146 d. 4578 e. 6123₈ f. 1155₈ Answer: 303 Answer: 3155 Answer: 621

DECIMAL TO OCTAL

One of the methods to convert decimal to octal is <u>by dividing the given decimal number</u> <u>recursively by 8</u>. Then, the remainders are noted down till we get 0 as the final quotient. After this step, these remainders are written in reverse order to get the octal value of the given decimal number.



Example 1: Convert 66₁₀ to an octal number

∴ 66₁₀ = 102₈

Example 2: Convert 103₁₀ to an octal number



• • •



MATH DRILLS!

Convert the followings decimal numbers into octal.

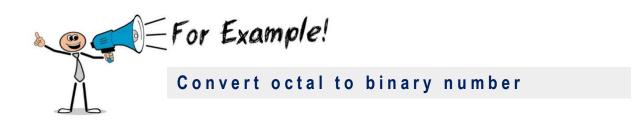
a. 18₁₀ **b.** 29₁₀ **c.** 46₁₀ Answer: 358 Answer: 228 Answer: 568 d. 73₁₀ e. 10510 f. 243₁₀ Answer: 111₈ Answer: 151₈ Answer: 3638

• • •

2.2 OCTAL TO BINARY & BINARY TO OCTAL

OCTAL TO BINARY

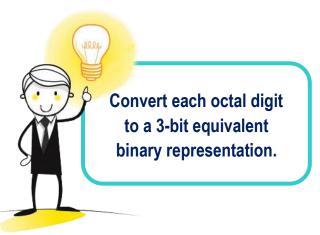
Each octal digit represents three (3) binary digits. Therefore, one (1) octal digit should give three (3) binary digits (bits).



Example 1: Convert 761₈ to a binary number.

	7		6			1		
2 ²	2 ¹	2 ⁰	2 ² 2 ¹ 2 ⁰ 2 ² 2 ¹					2 º
4	4 + 2 + 1			4 + 2 + 1			+2+	- 1
1	1	1	1	1	0	0	0	1

∴ 761₈ = 111110001₂



Example 2: Convert 2475₈ to a binary number.

2 4			7			5					
2 ²	2 ¹	2 ⁰	2 ²	2 ¹	2 ⁰	2 ²	2 ¹	2 ⁰	2 ²	2 ¹	2 ⁰
4	4 + 2 + 1 4 + 2 + 1		4 + 2 + 1		4 + 2 + 1						
0	1	0	1	0	0	1	1	1	1	0	1

 \therefore 2475₈ = 1 0 1 0 0 1 1 1 1 0 1₂



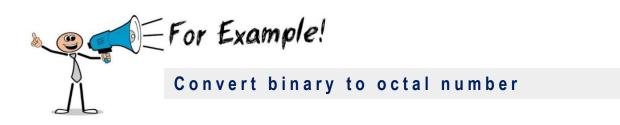
 MATH DRILLS!

 Convert the followings octal numbers into binary.

a.	468	b.	70 ₈	
	Answer: 100110 ₂			Answer: 111000 ₂
C.	6728	d.	155 ₈	
	Answer: 110111010 ₂	ſ	4550	Answer: 1101101 ₂
e.	10738	T.	1550 ₈	
	1			Amour 4404404000
	Answer: 1000111011 ₂			Answer: 11011010002

BINARY TO OCTAL

The binary number is grouped into a series of threes (3's) starting from the rightmost side. Then we must convert those groups into octal digit.



Example 1: Convert 11 010₂ to an octal number.

	1	1	0	1	0
2 ²	2 ¹	2 ⁰	2 ²	2 ¹	2 º
4 -	+ 2 -	+ 1	4 +	+ 2 -	+ 1
	3			2	



∴ 11 010₂ = 32₈

Example 2: Convert 10 101 111₂ to an octal number.

	1	0	1	0	1	1	1	1
2 ²	2 ¹	2 ⁰	2 ²	2 ¹	2 ⁰	2 ²	2 ¹	2 ⁰
4 + 2 + 1			4 +	- 2 -	+ 1	4 -	- 2 -	+ 1
	2			5			7	

∴ 10 101 111₂ = 257₈



MATH DRILLS!

Convert the followings binary numbers into octal.

a. 10110 ₂		b. 11111 ₂	
	Answer: 268		Answer: 378
c. 110011 ₂		d. 1010102	
	Answer: 638		Answer: 528
e. 1000001 ₂		f. 1111000 ₂	
	Answer: 101 ₈		Answer: 170 ₈
g. 10110000		h. 10100011	7 HOWEL 17 08
	Answer: 2608		Answer: 2438



03

HEXADECIMAL NUMBERING SYSTEM

3.1

- Hexadecimal to Decimal
- Decimal to Hexadecimal

3.2

- Hexadecimal to Binary
- Binary to Hexadecimal

3.3

- Hexadecimal to Octal
- Octal to Hexadecimal



3.1 HEXADECIMAL TO DECIMAL & DECIMAL TO HEXADECIMAL

HEXADECIMAL TO DECIMAL

To convert hexadecimal to decimal, we need to <u>multiply the hexadecimal digits with</u> <u>the power of 16</u> starting from the rightmost being in position 16^o.



Method 1:

Represent the number in terms of its positions.

```
(A \times 16^2) + (C \times 16^1) + (E \times 16^0)
= (10 x 256) + (12 x 16) + (14 x 1)
= 2560 + 192 + 14
= 2766<sub>10</sub>
```



Method 2:

Represent the number in terms of its positions.

 $E \times 16^{0} = 14 \times 1 = 14$ $C \times 16^{1} = 12 \times 16 = 192$ $A \times 16^{2} = 10 \times 256 = 2560$ 2766_{10}

∴ ACE₁₆ = 2766₁₀



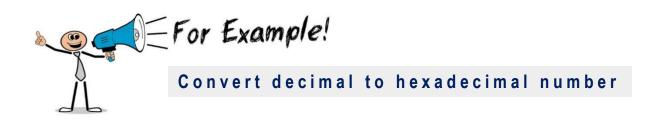
MATH DRILLS!

Convert the followings Base 16 numbers to Base 10.

a. 18A ₁₆	b. 2B9 ₁₆	c. C46 ₁₆
Answer: 394	Answer: 649	Answer: 3142
d. 73D1 ₁₆	e. E105 ₁₆	f. 24FA3 ₁₆
Answer: 29649 g. ABC ₁₆	Answer: 57605	Answer: 151459 i. 1BED ₁₆
Answer: 2748	Answer: 164	Answer: 7149

DECIMAL TO HEXADECIMAL

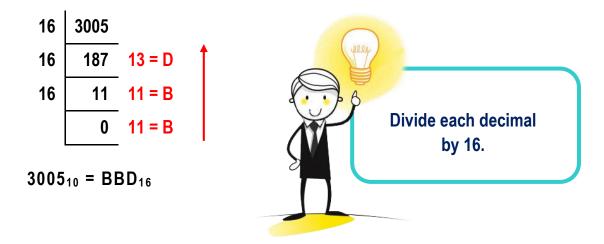
One of the methods to convert decimal to hexadecimal is <u>by dividing the given decimal</u> <u>number recursively by 16</u>. Then, the remainders are noted down till we get 0 as the final quotient. After this step, these remainders are written in reverse order to get the hexadecimal value of the given decimal number.



Example 1: Convert 465₁₀ to hexadecimal number.

....

Example 2: Convert 3005₁₀ to hexadecimal number.





MATH DRILLS!

Convert the followings Base 10 numbers to Base 16.

a. 48 ₁₀		b. 89 10		c. 141 ₁₀	
	Answer: 30 ₁₆		Answer: 59 ₁₆		Answer: 8D ₁₆
d. 333 ₁₀		e. 502 ₁₀)	f. 746 ₁₀	
g. 2022 ₁₀	Answer: 14D ₁₆	h. 1979	Answer: 1F6 ₁₆	i. 43 ₁₀	Answer: 2EA ₁₆
9					
	Answer: 7E6 ₁₆		Answer: 7BB ₁₆		Answer: 2B ₁₆
<u> </u>	AIISWEL ILU16				

3.2 **HEXADECIMAL TO BINARY & BINARY TO HEXADECIMAL**

HEXADECIMAL TO BINARY

Each hexadecimal digit represents four (4) binary digits. Therefore, one (1) hexadecimal digit should give four (4) binary digits (bits).



Example 1: Convert ACE₁₆ to a binary number.

	A =	10			C =	12		E = 14				
2 ³	2 ²	2 ¹	2 ⁰	2 ³	2 ²	2 ¹	2 ⁰	2 ³ 2 ² 2 ¹ 2 ⁰				
8	+4+	+ 2 +	2 + 1 8 + 4 + 2 + 1 8 + 4 +				+ 2 +	1				
1	0	1	0	1	1	0	0	1	1	1	0	



Convert each hexadecimal digit to a 4-bit equivalent binary representation.

Example 2: Convert 2B3D₁₆ to a binary number.

		3 B = 11			3				D = 13						
2 ³	2 ²	2 ¹	2 ⁰	2 ³	2 ²	2 ¹	2 ⁰	2 ³	2 ²	2 ¹	2 ⁰	2 ³	2 ²	2 ¹	2 º
8	+ 4 +	+ 2 +	1 8 + 4 + 2 + 1		1	8	8 + 4 + 2 + 1		· 1	8	+ 4 -	+ 2 +	1		
0	0	1	0	1	0	1	1	0	0	1	1	1	1	0	1

 $²B3D_{16} = 10\ 1011\ 0011\ 1101_2$...

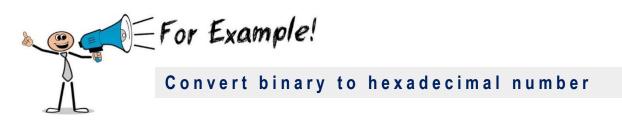


MATH DRILLS! Convert the followings Base 16 numbers to Base 2.

a. 46 ₁₆		b. 70D ₁₆
	Answer: 100 0110 ₂	Answer: 111 0000 1101 ₂
c. 6A2 ₁₆		d. 1EB ₁₆
	Answer: 110 1010 00102	Answer: 1 1110 1011 ₂
e. 10A ₁₆		f. 1500CC ₁₆
	Answer: 1 0000 10102	Answer: 1 0101 0000 0000 11002

BINARY TO HEXADECIMAL

The binary number is grouped into a series of fours (4's) starting from the rightmost side. Then we must convert those groups into hexadecimal digit.



Example 1: Convert 11 1011 1011₂ to hexadecimal number.

		1	1	1	0	1	1	1	0	1	1	
2 ³	2 ²	2 ¹	2 ⁰	2 ³	2 ²	2 ¹	2 ⁰	2 ³	2 ²	2 ¹	2 ⁰	
8	8 + 4 + 2 + 1			8	+ 4 +	+ 2 +	· 1	8 + 4 + 2 + 1				
	3				1	1			1	1		
	3	3			E	3			E	3		

```
∴ 11 1011 1011<sub>2</sub> = 3BB<sub>16</sub>
```



Example 2: Convert 1111 0010 1100 0111₂ to hexadecimal number.

1	1	1	1	0	0	1	0	1	1	0	0	0	1	1	1
2 ³	2 ²	2 ¹	2 ⁰	2 ³	2 ²	2 ¹	2 ⁰	2 ³	2 ²	2 ¹	2 ⁰	2 ³	2 ²	2 ¹	2 ⁰
8	8 + 4 + 2 + 1 8 + 4 + 2 + 1				8 + 4 + 2 + 1			8 + 4 + 2 + 1							
	3 2			12				7	7						
F				2	2			()		7				

.. 1111 0010 1100 0111₂ = F2C7₁₆

• • •



MATH DRILLS!

Convert the followings Base 2 numbers to Base 16.

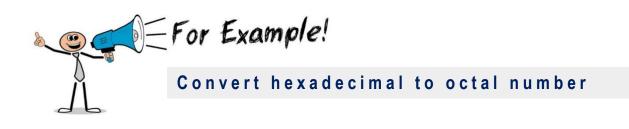
a. 1 0110 1101 ₂	b. 1	1111 1001 ₂	
	er: 16D16		Answer: F9 ₁₆
c. 11 0011 00102	d. 1	1 0101 0111 ₂	
Answ	er: 332 ₁₆		Answer: 157 ₁₆
e. 100 0001 00102	f. 1	111 1000 01112	
	er: 412 ₁₆		Answer: 787 ₁₆
g. 10 1100 0011 01012	h. 1	1 0100 0110 1010 ₂	
Answe	:: 2C3516		Answer: 146A ₁₆

3.3 HEXADECIMAL TO OCTAL & OCTAL TO HEXADECIMAL

HEXADECIMAL TO OCTAL

Use binary as an intermediary

- 1. Convert each hexadecimal digit a 4-bit equivalent binary representation
- 2. Group bit in 3's, starting on right
- 3. Convert to octal digit



Example 1: Convert BA5₁₆ to an octal number.

	B = 11				A =	10		5			
8 + 4 + 2 + 1 8 + 4 +				+ 2 +	2 + 1 8 + 4 + 2 + 3			- 1			
1	0	1	1	1	0	1	0	0	1	0	1
4	4 + 2 + 1 4 -			+2+	- 1	4 + 2 +		- 1	4 ·	+ 2 +	+ 1
5			6			4			5		

∴ BA5₁₆ = 5645₈

Example 2: Convert F4CE₁₆ to an octal number.

	F = 15			4				C = 12				E = 14			
8	8+4+2+1			8	+4·	+ 2 +	- 1	8 + 4 + 2 + 1			8 + 4 + 2 + 1				
1	1 1 1 1 0 1 0 0		1	1	0	0	1	1	1	0					

First, change the number to binary system:

Then, group the binary digit into 3 bits per group:

0	0		1	1	1	1	0	1	0	0	1	1	0	0	1	1	1	0
4	+ 2	+	1	4 ·	+ 2 +	+1	4 ·	+ 2 +	- 1	4 ·	+ 2 +	- 1	4 ·	+ 2 +	- 1	4 ·	+ 2 +	-1
	1				7			2			3			1			6	

∴ F4CE₁₆ = 172316₈



a. 25 ₁₆		b. 3D1 ₁₆	
	Answer: 458		Answer: 1721 ₈
c. 6A13 ₁₆		d. E48B ₁₆	
	Answer: 650238	(5500	Answer: 162213 ₈
e. ADAB ₁₆		f. FB20 ₁₆	
	Answer: 1266538		Answer: 1754408

• • • 44

OCTAL TO HEXADECIMAL

Use binary as an intermediary

- 1. Convert each octal digit a 3-bit equivalent binary representation
- 2. Group bit in 4's, starting on right
- 3. Convert to hexadecimal digit



Example 1: $107_8 \rightarrow ____{16}$

	1			0		7			
4 + 2 + 1			4 + 2 + 1			4 + 2 + 1			
0	0	1	0	0 0 0		1	1	1	
	8 + 4 + 2 + 1				8 + 4 + 2 + 1				
		4	1			7	7		

∴ **107**₈ = 47₁₆

Example 2: $1076_8 \rightarrow ____{16}$

1			0			7		6			
4 ·	4 + 2 + 1 4			+ 2 + 1 4 + 2 -			+ 1 4 + 2 + 1			- 1	
0	0	1	0	0	0	1	1	1	1	1	0
8 + 4 + 2 + 1			8 + 4 + 2 + 1			1	8	+ 4 -	+ 2 +	1	
2					3			14	= E		

∴ 1076₈ = 23E₁₆



a. 2468		b. 577 ₈	
	Answer: A616		Answer: 17F ₁₆
c. 6720 ₈		d. 31538	
	Answer: DD016		Answer: 66B ₁₆
e. 1331₃		f. 11228	
	Answer: 2D9 ₁₆		Answer: 25216

TABLE: CONVERSION OF NUMBER SYSTEM

From To	DECIMAL	BINARY	OCTAL	HEXADECIMAL
DECIMAL		Divide by 2	Divide by 8	Divide by 16
BINARY	Multiply each bit by 2 ⁿ		 Group bit in 3's, starting on right. Convert to octal digit 	 Group bit in 4's, starting on right. Convert to hexa. digit
OCTAL	Multiply each bit by 8 ⁿ	Convert each octal digit to a 3-bit equivalent binary representation		Use binary as an intermediary
HEXADECIMAL	Multiply each bit by 16 ⁿ	Convert each hexa. digit to a 4-bit equivalent binary representation	Use binary as an intermediary	





BINARY ARITHMETIC

4.1

Binary Arithmetic Operations Addition, Subtraction and Multiplication



4.1 **BINARY ARITHMETIC OPERATIONS**

BINARY ADDITION

Rules:

Α	В	A + B
0	0	0
0	1	1
1	0	1
1	1	0 and carry 1 to the next more significant bit



Example 1:

	1				1		carries
		1	0	1	0	1	
+		1	1	0	0	1	
	1	0	1	1	1	0	

Example 2:

			1	1		carries
	1	0	0	1	1	
+		1	0	1	1	
	1	1	1	1	0	_
						_





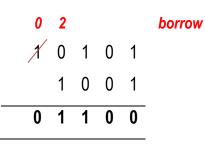
a. 1101 + 1000		b. 1111 + 1010	
	Answer: 10101		Answer: 11001
c. 1100110 + 100		d. 1010101 + 11010	
	nswer: 1101010		Answer: 1101111
e. 100110 + 100101		f. 1111111 + 100010	
Ą	nswer: 1001011		Answer: 10100001

BINARY SUBTRACTION

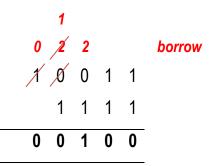
Rules:

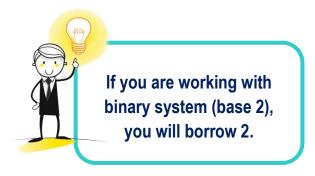
Α	В	A - B
0	0	0
0	1	1 and borrow 1 from the next more significant bit
1	0	1
1	1	0

Example 1:



Example 2:









a. 1101 – 1000		b. 1111 – 1010	
- 1100110 100	Answer: 101	d 4040404 44040	Answer: 101
c. 1100110 – 100		d. 1010101 – 11010	
	Answer: 1100010		Answer: 111011
e. 1110001 – 10111		f. 10000000 – 100010)1
	Answer: 1011010		Answer: 111011

BINARY MULTIPLICATION

Rules:

Α	В	A x B
0	0	0
0	1	0
1	0	0
1	1	1



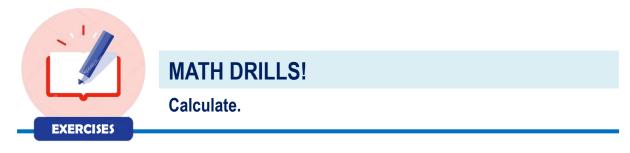
Example 1:

Example 2:

	1	0	0	1	1	0	1	0
+		1	1	1	0			
			0	0	0	0		
				1	1	1	0	
					1	1	1	0
×					1	0	1	1
					1	1	1	0

	1	1	1	0	0
×			1	0	0
			1	1	1





a.	1101 × 100		b.	1111 :	× 101	
•	11001 1000	Answer:11 0100	4	10101	× 1101	Answer:100 1011
C.	11001 × 1000		a.	10101	X IIVI	
		Answer:1100 1000				Appwor:1 0001 0001
e.	11001 × 1110	Answer. 1100 1000	f.	10000	× 1010	Answer:1 0001 0001
		Answer: 1 0101 1110				Answer: 1010 0000



05

NUMBER BASE CONVERSION USING SCIENTIFIC CALCULATOR

5.1

Convert One Base to Another



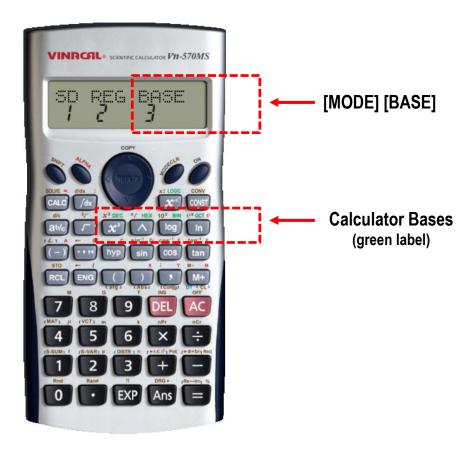
• • • 55

5.1 CONVERT ONE BASE TO ANOTHER

Before starting the calculation, you must first enter the correct mode related to the type of calculation you want to perform. The number base can be called from the calculator using the following mode operation [MODE] [BASE]. The calculator can be used to calculate the following bases: Decimal (Base 10), Binary (Base 2), Octal (Base 8) and Hexadecimal (Base 16).

Basically, calculator bases represented as **BIN**-Binary, **DEC**-Decimal, **OCT**-Octal and **HEX**-Hexadecimal. The green label is reserved for basic number calculations. This allows you to add / subtract / divide and multiply in different number bases.

The calculation mode can be returned to the initial default by pressing the [MODE] [COMP].



Calculator

CONVERT BINARY TO DECIMAL

Example 1: Convert 10011₂ to decimal.

Procedure:

- 1. Press [MODE] [BASE] [BIN] [1001] [=] [DEC] (Result: 19).
- 2. Thus, the result is 19 in decimal.

CONVERT OCTAL TO DECIMAL

Example 2: Convert 2467₈ to decimal.

Procedure:

- 1. Press [MODE] [BASE] [OCT] [2467] [=] [DEC] (Result: 1335).
- 2. Thus, the result is 1335 in decimal.

CONVERT HEXADECIMAL TO DECIMAL

Example 3: Convert F14E₁₆ to decimal.

Procedure:

- 1. Press [MODE] [BASE] [HEX] [F14E] [=] [DEC] (Result: 61774).
- 2. Thus, the result is 61774 in decimal.

CONVERT DECIMAL TO HEXADECIMAL

Example 4: Convert 47818₁₀ to hexadecimal.

Procedure:

- 1. Press [MODE] [BASE] [DEC] [47818] [=] [HEX] (Result: BACA).
- 2. Thus, the result is BACA in hexadecimal.

CONVERT DECIMAL TO OCTAL

Example 5: Convert 47818₁₀ to octal.

Procedure:

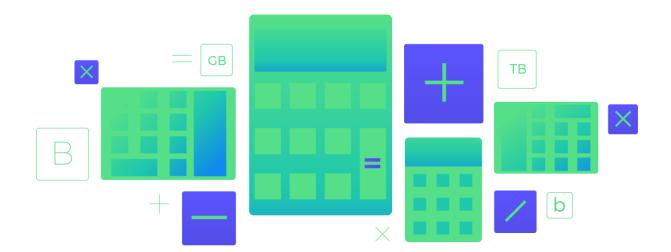
- 1. Press [MODE] [BASE] [DEC] [47818] [=] [OCT] (Result: 135312).
- 2. Thus, the result is 135312 in octal.

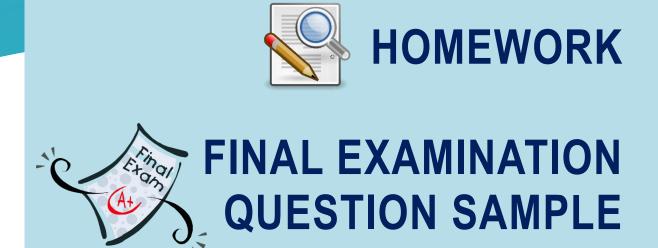
CONVERT DECIMAL TO BINARY

Example 6: Convert 468₁₀ to binary.

Procedure:

- 1. Press [MODE] [BASE] [DEC] [468] [=] [BIN] (Result: 111010100).
- 2. Thus, the result is 111010100 in binary.







HOMEWORK

- 1. Convert the binary numbers into decimal numbers:
 - a) 11
 - b) 101
 - c) 1111
 - d) 110111011
 - e) 1111100011110011

2. Convert the decimal numbers into binary numbers:

- a) 7
- b) 13
- c) 128
- d) 1678
- e) 12359
- 3. Convert the following binary numbers to hexadecimal (base-16):
 - a) 0111
 - b) 1101
 - c) 1011011
 - d) 11101100
 - e) 100101101
 - f) 0101101011110000

 - h) 11001010111001101011010011
- 4. Fill in the blanks:

Decimal	Binary	Octal	Hexadecimal
33			
	1110101		
		703	
			1AF

- 5. Convert the following hex numbers to binary:
 - a) DEAD
 - b) AF051257
 - c) C9FF2A9
 - d) 5E5E
 - e) 123456789
 - f) ABCDEF
 - g) B0C
 - h) 1010
- 6. Convert the following octal (base-8) numbers to decimal:
 - a) 4
 - b) 10
 - c) 777
 - d) 05726
 - e) 183242
- 7. Convert the following decimal numbers to octal:
 - a) 5
 - b) 9
 - c) 625
 - d) 1,024
 - e) 32,767
- 8. Add the following 4-bit binary numbers:
 - a) 1011₂ + 1001₂
 - b) 1100₂ + 0110₂
 - c) 1010₂ +0011₂
 - d) 1101₂ +1011₂
 - e) 1001₂ + 0111₂
 - f) 1100₂ + 1001₂

HOMEWORK

• • •

- 9. Add the following 8-bit binary numbers:
 - a) $11001010_2 + 10011010_2$
 - b) $10011100_2 + 01111001_2$
 - c) 11001011₂ + 10000011₂
 - d) $11100001_2 + 10011101_2$
 - e) $1000001_2 + 01100110_2$
 - f) $10010011_2 + 10000111_2$

10. Calculate the binary numbers:

a) 11 – 10	b) 110 – 10	c) 1111 – 110	d) 100 – 10
e) 100 – 11	f) 1000 – 11	g) 1101 – 110	h) 11011 – 110
i) 1111 – 111	j) 110101 – 1010	k) 11011 – 111	l) 11110 – 111

- 11. Subtract the following 4-bit binary numbers:
 - a) 1011₂ 1001₂
 - b) 1100₂ 0110₂
 - c) 1010₂ 0011₂
 - d) 1101₂ 1011₂
 - e) 1001₂ 0111₂
 - f) 1100₂ 1001₂
- 12. Multiply the following 3-bit numbers:
 - a) 100₂ * 010₂
 - b) 111₂ * 010₂
 - c) 110₂ * 011₂
 - d) 101₂ * 011₂
 - e) 110₂ * 101₂
 - f) 101₂ * 101₂

HOMEWORK

. . .

13. Calculate:

- a) 150₈ + 75₈
- b) $532_8 174_8$
- c) 423₈ + 326₈
- d) $423_8 326_8$
- e) 56712₈ + 24415₈
- f) $56712_8 24415_8$
- g) 1023₈ + 424₈
- h) $1023_8 424_8$
- i) 354₈ + 622₈
- j) $1103_8 243_8$

14. Calculate:

- a) $36_{16} + 4D_{16}$
- b) 4D₁₆ + DE₁₆
- c) $DE_{16} 7B_{16}$
- d) 7B₁₆ + 6A₁₆
- e) 6A₁₆ + 85₁₆
- f) $DF_{16} 11_{16}$
- g) $BD_{16} 11_{16}$
- h) BA3₁₆ + 5DE₁₆

-End of Homework-



FINAL EXAMINATION

QUESTION SAMPLE

- 1. Complete the following in terms of data organization that stated in bracket.
 - a) 24 bits (nibbles)
 - b) 5 words (bits)
 - c) 128 bytes (double words)
- 2. Convert the following:
 - a) 11100011_2 to decimal
 - b) 62_8 to binary
 - c) 444₈ to hexadecimal
 - d) 2017₁₀ to octal
 - e) 33₁₀ to binary
- 3. Calculate the following:
 - a) 1101₂ + DAD1₁₆ (Give your answer in octal)
 - b) $246_8 59_{10}$ (Give your answer in hexadecimal)

-End of Final Examination Question Sample-



REFERENCES

Abdul Hadi Yaakub, Ong Beng Sim et al. (2012). *Mathematics for Matriculation Semester* 2: Fourth Edition Update. Kuala Lumpur: Fajar Bakti Sdn. Bhd.

Hanrahan, V. (2013). OCR Additional Mathematics Practice Book. London: Hodder Education.

O'Regan, G. (2012). *Mathematics in Computing: An Accessible Guide to Historical, Foundation and Application Contexts*. New York: Springer.

FEEDBACK





"This book is one of the mathematical references that covers a solid understanding of the number system. In this book, we will explore the binary, octal, hexadecimal and decimal number systems as well as the addition, subtraction and multiplication operations of the number system. This book also includes easy to -understand explanations and exam -shaped questions."

> by: Normi Ismail | Melati Sabtu Department of Mathematics, Science and Computer Politeknik Kuala Terengganu, Terengganu

