

STATISTICS AND PROBABILITY

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TEKNOLOGI & PERKOMPUTERAN

STATISTICS AND PROBABILITY

First Published 2021
Published 2021 (electronic)

Politeknik Kuala Terengganu

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STATISTICS AND PROBABILITY

Published by :

Politeknik Kuala Terengganu,
20200 Jalan Sultan Ismail,
Kuala Terengganu,
Terengganu.

Perpustakaan Negara Malaysia

Cataloguing-in-Publication Data

Salihah Zakaria, 1983-

STATISTICS AND PROBABILITY / Penulis: SALIHAN BT ZAKARIA.

Mode of access: Internet eISBN 978-967-2240-21-1

Statistics.

Probabilities.

Government publications--Malaysia.

Electronic books.

I. Title. 519.5

INTRODUCTION

STATISTICS AND PROBABILITY is written for students studying Engineering Mathematics in local polytechnic institutions. It is based on the current syllabus of the Malaysian polytechnic. This book as a resources and references for students gain better understanding about the statistical and probability concepts and their applications in interpreting data.

ACKNOWLEDGEMENTS

I would like to thank for everybody involved in the production of this book. Special thanks to my family and colleagues for their support and encouragement throughout the preparation for this book.

Finally, I would like to express my gratitude to the publisher and for all involved in the production of book. I hope this book would serve its purpose in helping students gain better understanding of the statistical and probability concepts.

Salihah Zakaria

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DEMONSTRATE STATISTICAL DATA

1.1 Demonstrate statistical data

Statistics represent scientific procedures and methods for collecting, organizing, summarizing, presenting and analyzing as well as obtaining useful information, drawing valid conclusions and making effective decisions based on analysis.

Data is a collection of information, measurement or observation obtained from study that is carried out. Normally, data is in the form of numbers. The data can be about marks scored by a class, death, population, etc. Data can be categorized into two types. They are quantitative data and qualitative data.



Quantitative data are data that can be measured numerically. It is also known as numerical data. They are two types of quantitative data – Discrete data and Continuous data.

(a) **Discrete data** are data that have exact values and can be counted.

Example : A number of children in family.

(b) **Continuous data** are data do not have exact values and it is found through measurement.

Example: The weight of babies in a clinic.

Qualitative data are data that cannot assume a numerical value but can be classified into two or more categories. It is also known as attributive data.

Example: Gender, race, blood type, colour, etc

Example 1

State the type of data for each

- The height of students
- The length of table
- The number of cats
- The weight of banana
- The number of books

Solution

- | | |
|--------------------|--------------------|
| a) Continuous data | d) Continuous data |
| b) Continuous data | e) Discrete data |
| c) Discrete data | |

DEMONSTRATE STATISTICAL DATA

1.1.1 Define statistical terminology of ungrouped data and grouped data

Ungrouped Data

Raw data is data which have not been organized numerically. When these data are arranged, a set of organized data is acquired.

Frequency Table

- a) Frequency refers to the number of times an event or a value occurs.
- b) A frequency table is a table that lists items and shows the number of times the items occur
- c) We represent the frequency by the English alphabet ‘f’.
- d) Tally marks help to record frequency of occurrence of something.



Creating a frequency table

Step 1: Make three columns. The first column carries the data values in ascending order

Step 2: The second column contains the number of times the data value occurs using tally marks. Count for every row in the table. Use tally marks for counting.

Step 3: Count the number of tally marks for each data value and write it in the third column

Example 2

The table below shows the marks obtained by 30 students in a quiz.

4	7	4	5	7
9	6	7	10	9
5	4	10	5	5
8	10	7	7	8
6	7	7	10	5
10	5	9	8	8

DEMONSTRATE STATISTICAL DATA

Solution

MARKS	TALLY	FREQUENCY
4	III	3
5	HHII	6
6	II	2
7	HHI II	7
8	IIII	4
9	III	3
10	HHI	5



Grouped Data is data that is organized and arranged into different categories. For set having more than ten members, those members having similar values are grouped together in classes to form a frequency table.

A frequency table is merely a table showing categories and classes and their corresponding frequencies. The new set of values obtained by forming a frequency table is called grouped data.

Grouped data can be presented diagrammatically in a few ways. They are histogram, frequency polygon and ogive.

Creating Frequency Table (from ungrouped data to grouped data)

1. Determine the number of data
2. Determine the value of highest value of data and lowest value of data
3. Find the range

$$\text{Range} = \text{Highest value of data} - \text{lowest value of data}$$

4. Find the number of classes

$$k = 1 + 3.3 \log N$$

5. Find the size of class interval, C

$$\text{Size of class interval, } c = \frac{\text{Range}}{\text{Number of classes}}$$

6. Construct the frequency table

DEMONSTRATE STATISTICAL DATA

Example 3

The table below shows the marks obtained by 30 students from class A in a mathematics test.

45	66	57	42	56
47	54	65	62	53
60	65	43	58	44
40	60	58	63	68
50	54	51	51	52
56	50	47	61	53

- Find the number of classes, k
- Determine the data range
- Determine the class width, C
- Construct a frequency distribution table

Solution

$$\begin{aligned} \text{a) } k &= 1 + 3.3 \log N \\ &= 1 + 3.3 \log 30 \\ &= 1 + 4.8745 \\ &= 5.8745 \sim 6 \end{aligned}$$

$$\begin{aligned} \text{b) Data range} &= 68 - 40 \\ &= 28 \end{aligned}$$

$$\begin{aligned} \text{c) Class width, } C &= \frac{\text{Range}}{\text{Number of classes}} \\ &= \frac{28}{6} \\ &= 4.6667 \sim 5 \end{aligned}$$

d) Frequency distribution table

Class Interval	Frequency
40-44	4
45-49	3
50-54	9
55-59	5
60-64	5
65-69	4



DEMONSTRATE STATISTICAL DATA

Example 4

A sample of 25 newborn babies is collected from a hospital. The masses of the newborn babies are given in the table below.

3.1	3.0	3.3	2.8	3.2
3.4	2.9	3.4	2.9	2.3
3.1	3.0	1.7	3.1	2.0
1.8	3.4	2.9	2.6	2.5
2.7	3.5	2.4	2.8	3.2



- Find the number of classes, k
- Determine the data range
- Determine the class width, C
- Construct a frequency distribution table

Solution

- a) The number of classes, k

$$\begin{aligned}k &= 1 + 3.3 \log N \\&= 1 + 3.3 \log 25 \\&= 1 + 4.6132 \\&= 5.6132 \\&\sim 6\end{aligned}$$

- b) *Data range* = $3.5 - 1.6$
= 1.7

- c) Class width, $C = \frac{\text{Range}}{\text{Number of classes}}$
= $\frac{1.8}{6}$
= 0.3

DEMONSTRATE STATISTICAL DATA

d) Frequency distribution table

Class Interval	Frequency
1.7-1.9	2
2.0-2.2	1
2.3-2.5	3
2.6-2.8	4
2.9-3.1	8
3.2-3.4	6
3.5-3.7	1



Histogram

A histogram is a graphical representation of the frequency distribution in which bars represent frequencies. The histogram is constructed by using class boundaries and frequencies of the classes. The frequency is represented by the area of the bar. The area is equivalent to the height of the bar for equal class intervals. The frequency is represented by the area of the bar. The area is equivalent to the height of the bar for equal class intervals.

When plotting histograms, the random variable or phenomenon of interest is plotted along the horizontal axis; the vertical axis represents the number, proportion, or percentage of observations per class interval, depending on whether the particular histogram is a frequency histogram, a relative frequency histogram, or a percentage histogram respectively.

Steps to construct a histogram

- Identify class boundaries and frequency for each class.
- Mark the class boundaries at the horizontal axis (x)
- Insert the frequency at vertical axis (y)
- Draw a vertical bar to show the frequency for each class.

DEMONSTRATE STATISTICAL DATA

Example 5:

The following table shows the heights (in cm) distribution of 30 workers.

Heights	Number of workers
140-149	6
150-159	9
160-169	7
170-179	5
180-189	2
190-199	1



Solution

Heights	Number of workers	Class boundaries
140-149	6	139.5-149.5
150-159	9	149.5-159.5
160-169	7	159.5-169.5
170-179	5	169.5-179.5
180-189	2	179.5-189.5
190-199	1	189.5-199.5

The height (in cm) distribution of workers

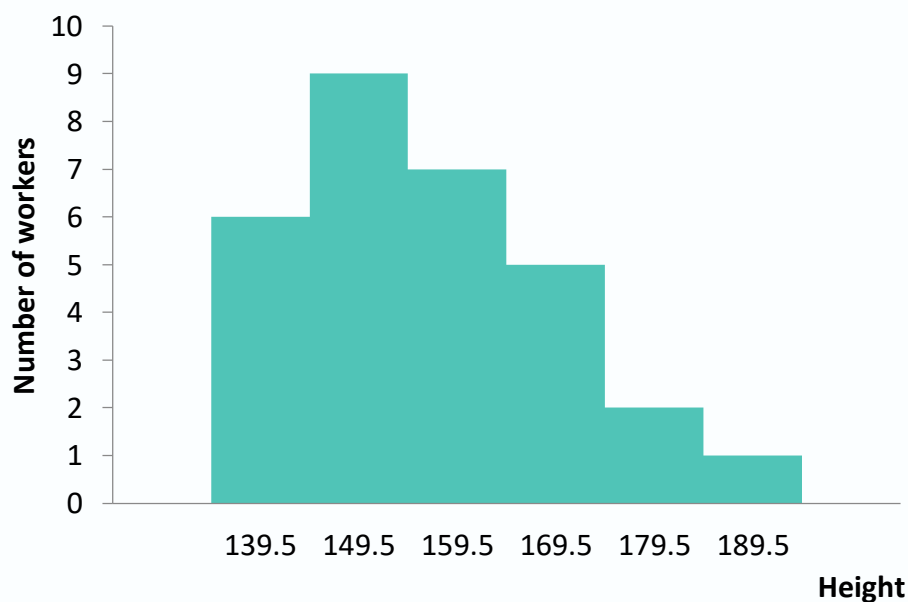


Figure 1 A histogram for the heights(in cm) distribution of 30 workers

DEMONSTRATE STATISTICAL DATA

Ogive

An ogive is also known as the cumulative frequency curve. It is a graph of a cumulative frequency distribution.

There are two types of ogives.

a) 'less than' ogive.

'Less than' ogive is an increasing function. It rises to the right.

a) The 'more than' ogive

'More than' ogive falls to the right.



Steps to draw ogive

- Add an extra class with 0 frequency before the initial class
- Find the class boundaries and create a 'more than' cumulative frequency
- Draw an ogive on the graph sheet with lower class boundaries on x-axis and cumulative frequency on y-axis.

Example 6

The speed of 100 cars that pass a highway during a certain period are shown in the following table

Speed (km/h)	Number of cars
55-59	5
60-64	35
65-69	29
70-74	15
75-79	11
80-84	5

Draw a 'less than ogive and more than ogive to present the speed

DEMONSTRATE STATISTICAL DATA

Solution

a) Less than ogive

Speed (km/h)	Number of cars	Class boundaries (less than)	Cumulative frequency (less than)
50-54	0	54.5	0
55-59	5	59.5	5
60-64	35	64.5	40
65-69	29	69.5	69
70-74	15	74.5	84
75-79	11	79.5	95
80-84	5	84.5	100



A less than ogive the speed of 100 cars

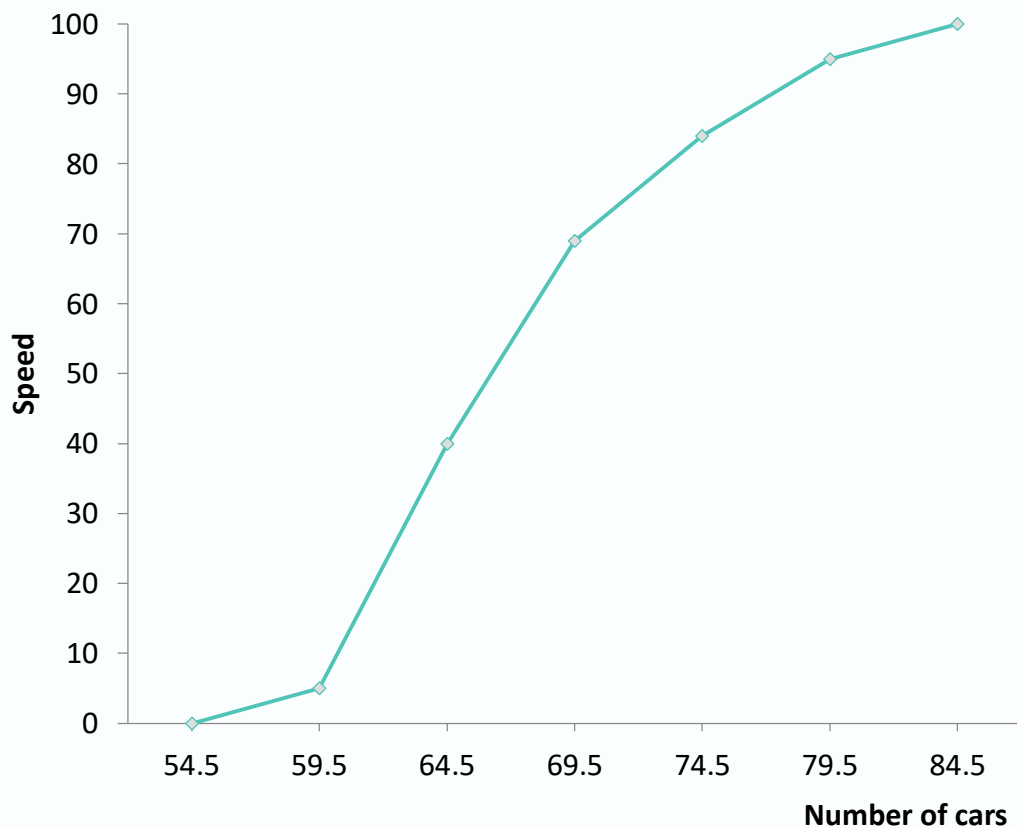


Figure 2 The ogive for the speed of 100 cars that pass a highway during a certain period

DEMONSTRATE STATISTICAL DATA

b) More than ogive

Speed (km/h)	Number of cars	Class boundaries (More than)	Cumulative frequency (More than)
50-54	0	54.5	100
55-59	5	59.5	95
60-64	35	64.5	60
65-69	29	69.5	31
70-74	15	74.5	16
75-79	11	79.5	5
80-84	5	84.5	0

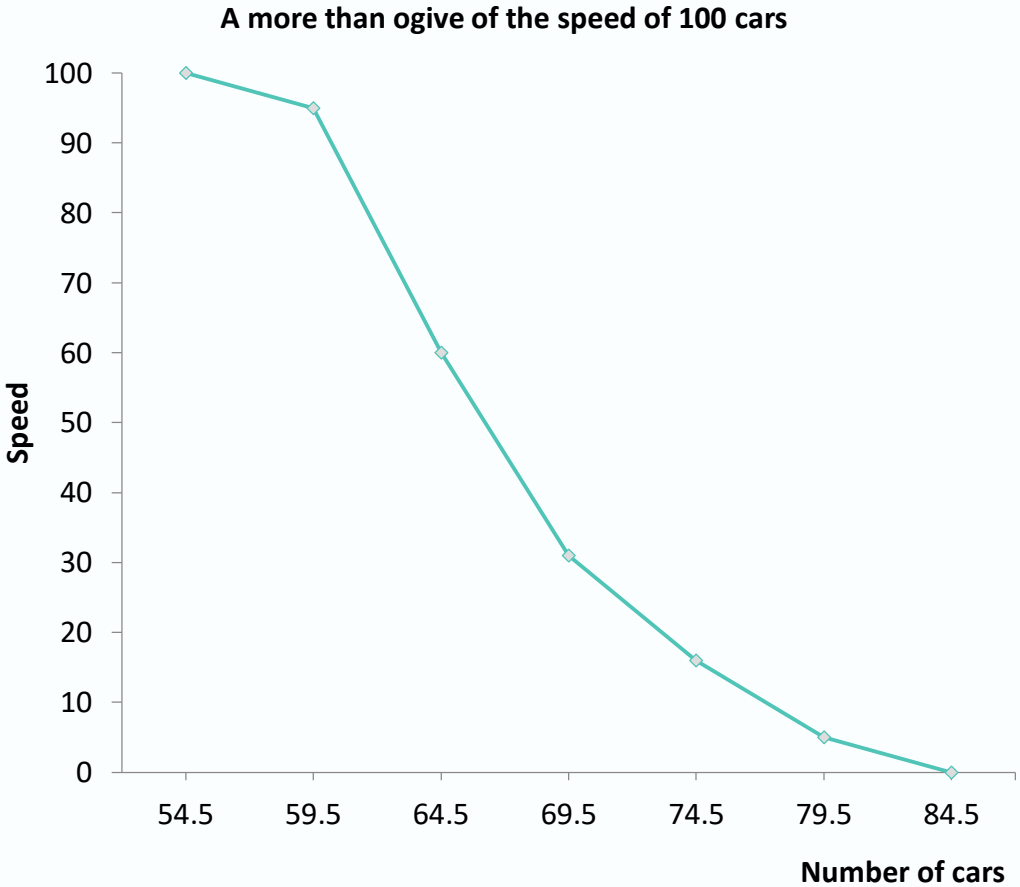


Figure 3 The ogive for the speed of 100 cars that pass a highway during a certain period

Exercise 1.1

1. Determine whether the data given below is continuous or discrete

- The number of siblings in a family.
- The weight of watermelon in a basket
- The height of patients in a clinic
- The number of tables in a classroom
- The number of workers in a company.

2. The data below shows height (in cm) of 30 workers of Company ABC.

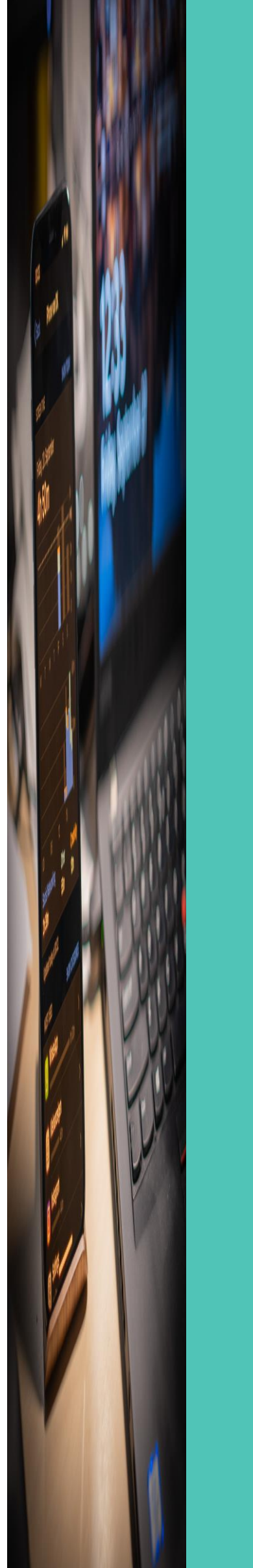
178	185	189	181	187
166	170	171	173	182
170	176	168	175	175
180	164	184	163	175
171	176	179	169	187
183	172	165	163	188

Construct a frequency table which contains 5 classes.

3. A set of data is shown as follows

9	6	5	7	7
4	4	6	8	8
7	7	7	8	6
8	10	5	9	6

Construct a frequency table



COMPUTE OF CENTRAL TENDENCY AND DISPERSION

1.2 Compute of central tendency and dispersion

Measures of **central tendency** are the number used to represent the center of middle of a set of data values. It is also called as the average.

The three common methods of finding the average are:

1. The mean
2. The median
3. The mode



Measures of **dispersion** indicates the scattering of data. The measures of dispersion of data are range, quartile, decile, mean deviation, variance and standard deviation

1.2.1 Calculate mean, median and mode for ungrouped data.

Mean

- i. Mean can be calculated by summing up the values of data divided by the number of data.

$$\text{Mean, } \bar{x} = \frac{\sum x}{n}$$

$\sum x$ = sum of all the data

n = number of data

Median

- i. Median is the value in the middle of a set of data after the value of data are arranged in ascending order.

Step 1: Arrange the data in an ascending order.

Step 2: Determine the location of median.

Step 3: Find the value of median.

Mode

- i. Mode is the most commonly occurring value in a set of data.

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

Example 7

A set of data is shown as follows

6, 7, 2, 5, 10, 10, 3, 1, 6, 10

Find the,

- Mean, \bar{x}
- Median, m
- Mode

Solution

- Mean, \bar{x}

$$\begin{aligned}\bar{x} &= \frac{\sum x}{n} \\ &= \frac{6+7+2+5+10+10+3+1+6+10}{10} \\ &= \frac{60}{10} \\ &= 6\end{aligned}$$

- Median, m

i. Arrange the data in ascending order

1, 2, 3, 5, 6, 6, 7, 10, 10, 10

ii. Determine the location of median

$$\begin{aligned}\frac{n+1}{2} &= \frac{10+1}{2} \\ &= 5.5^{\text{th}} \text{ observation}\end{aligned}$$

iii. The value of median = $\frac{6+6}{2}$
= 6

- Mode

1, 2, 3, 5, 6, 6, 7, 10, 10, 10

$$m = 10$$



COMPUTE OF CENTRAL TENDENCY AND DISPERSION

Example 8

The table below shows the number of shirts in the 40 baskets.

Shirts	Frequency
25	2
26	5
27	8
28	11
29	10
30	4

Calculate

- The Mean, \bar{x}
- The Median, m
- The Mode

Solution

- The Mean, \bar{x}

Shirts, x	Frequency, f	fx
25	2	50
26	5	130
27	8	216
28	11	308
29	10	290
30	4	120

$$\begin{aligned} \text{Mean, } \bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{1114}{40} \\ &= 27.85 \text{ shirts} \end{aligned}$$

$\sum f = 40$ $\sum fx = 1114$



COMPUTE OF CENTRAL TENDENCY AND DISPERSION

b. The Median, m

Shirts, x	Frequency, f	Cumulative frequency	Position of data
25	2	2	1-2
26	5	7	3-7
27	8	15	8-15
28	11	26	16-26 *
29	10	36	27-36
30	4	40	37-40

Median, m = Location of median

$$= \frac{n+1}{2}$$

$$= \frac{40+1}{2}$$

$$= 20.5$$

Value of median = 28 shirts

c. The Mode

= 28 (The value that occurs most frequently in a data)

1.2.2 Calculate mean, median and mode for grouped data by using formula.

$$\text{Mean, } \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$\sum f_i x_i$ = sum of the frequency \times midpoint

$\sum f$ = sum of frequency

$i = 1, 2, 3, \dots, n$

$$\text{Median, } m = L_m + \left(\frac{\frac{N}{2} - f}{f_m}\right)c$$

L_m = Lower boundary of the class in which the median lies.

n = Total frequency

f = Cumulative frequency **before** the class in which the median lies.

f_m = Frequency of the class in which the median lies.

c = class size



COMPUTE OF CENTRAL TENDENCY AND DISPERSION

$$\text{Mode, } m_o = L_{mo} + \left(\frac{d_1}{d_1 + d_2}\right)c$$

where

L_{mo} = Lower boundary of the class in which the mode lies

d_1 = Difference between the **frequency of mode class** and **class before it**.

d_2 = Difference between the **frequency of mode class** and **class after it**.

c = Class size



Example 1.9

1. The heights of 40 students are recorded in the table below.

Height (in cm)	No. of students
150-154	5
155-159	3
160-164	4
165-169	11
170-174	9
175-179	8

Calculate

- Mean
- Median
- Mode

Solution

Height (in cm)	No. of students, f	Midpoint, x	Class boundaries	fx	Cumulative frequency	Position of data
150-154	5	152	149.5-154.5	760	5	1-5
155-159	3	157	154.5-159.5	471	8	6-8
160-164	4	162	159.5-164.5	648	12	9-12
165-169	11	167	164.5-169.5	1837	23	13-23
170-174	9	172	169.5-174.5	1548	32	24-32
175-179	8	177	174.5-179.5	1416	40	33-40
	40			6680		

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

a. Mean, $\bar{x} = \frac{\sum fx}{\sum f}$

$$= \frac{6680}{40}$$

$$= 167$$

b. Median, m

$$\text{Location of Median} = \frac{N}{2}$$

$$= \frac{40}{2}$$

$$= 20$$

Class interval for median = 165-169

$$\text{Median, } m = L_m + \left(\frac{\frac{N}{2} - f}{f_m}\right)c$$

$$= 164.5 + \left(\frac{\frac{40}{2} - 12}{11}\right)5$$

$$= 168.136$$

c. Mode

Class of interval of mode = 165-169 (the highest frequency)

$$\text{Mode, } m_o = L_{mo} + \left(\frac{d_1}{d_1 + d_2}\right)c$$

$$= 164.5 + \left(\frac{7}{7+2}\right)5$$

$$= 168.389$$

$$d_1 = 11 - 4$$

$$= 7$$

$$d_2 = 11 - 9$$

$$= 2$$



COMPUTE OF CENTRAL TENDENCY AND DISPERSION

Example 10

The table below shows a life content of 40 batteries.

Time (years)	Number of batteries
1.5-1.9	3
2.0-2.4	2
2.5-2.9	4
3.0-3.4	12
3.5-3.9	14
4.0-4.4	5

Calculate

- Mean
- Median
- Mode



Solution

Time (years)	Number of batteries, f	Midpoint, x	fx	Class boundaries	Cumulative frequency	Position of data
1.5-1.9	3	1.7	5.1	1.45-1.95	3	1-3
2.0-2.4	2	2.2	4.4	1.95-2.45	5	4-5
2.5-2.9	4	2.7	10.8	2.45-2.95	9	6-9
3.0-3.4	12	3.2	38.4	2.95-3.45	21	10-21
3.5-3.9	14	3.7	51.8	3.45-3.95	35	22-35
4.0-4.4	5	4.2	21.0	3.95-4.45	40	36-40
	40		131.5			

a. Mean, $\bar{x} = \frac{\sum fx}{\sum f}$

$$= \frac{131.5}{40}$$

$$= 3.288$$

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

b. Median, m

$$\begin{aligned}\text{Location of Median} &= \frac{N}{2} \\ &= \frac{40}{2} \\ &= 20\end{aligned}$$

Class interval for median = 3.0-3.4

$$\begin{aligned}\text{Median, } m &= L_m + \left(\frac{\frac{N}{2} - f}{f_m}\right)c \\ &= 2.95 + \left(\frac{20 - 9}{12}\right)0.5 \\ &= 3.408\end{aligned}$$



c. Mode

Class of interval of mode = 3.5-3.9

$$\begin{aligned}\text{Mode, } m_o &= L_{m_o} + \left(\frac{d_1}{d_1 + d_2}\right)c & d_1 &= 14 - 12 \\ & & &= 2 \\ &= 3.45 + \left(\frac{2}{2+9}\right)0.5 & d_2 &= 14 - 5 \\ & & &= 9 \\ &= 3.541\end{aligned}$$

1.2.3 Calculate median and mode for grouped data by using graph

Example 11

The height of 40 students are recorded in the table below.

Height (in cm)	Number of students
150-154	5
155-159	3
160-164	4
165-169	11
170-174	9
175-179	8

- Draw a histogram and determine the mode from the histogram.
- Draw an ogive and determine the median from the ogive.

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

Solution

a. The mode

Height (in cm)	Number. Of students	Lower boundaries
150-154	5	149.5
155-159	3	154.5
160-164	4	159.5
165-169	11	164.5
170-174	9	169.5
175-179	8	174.5

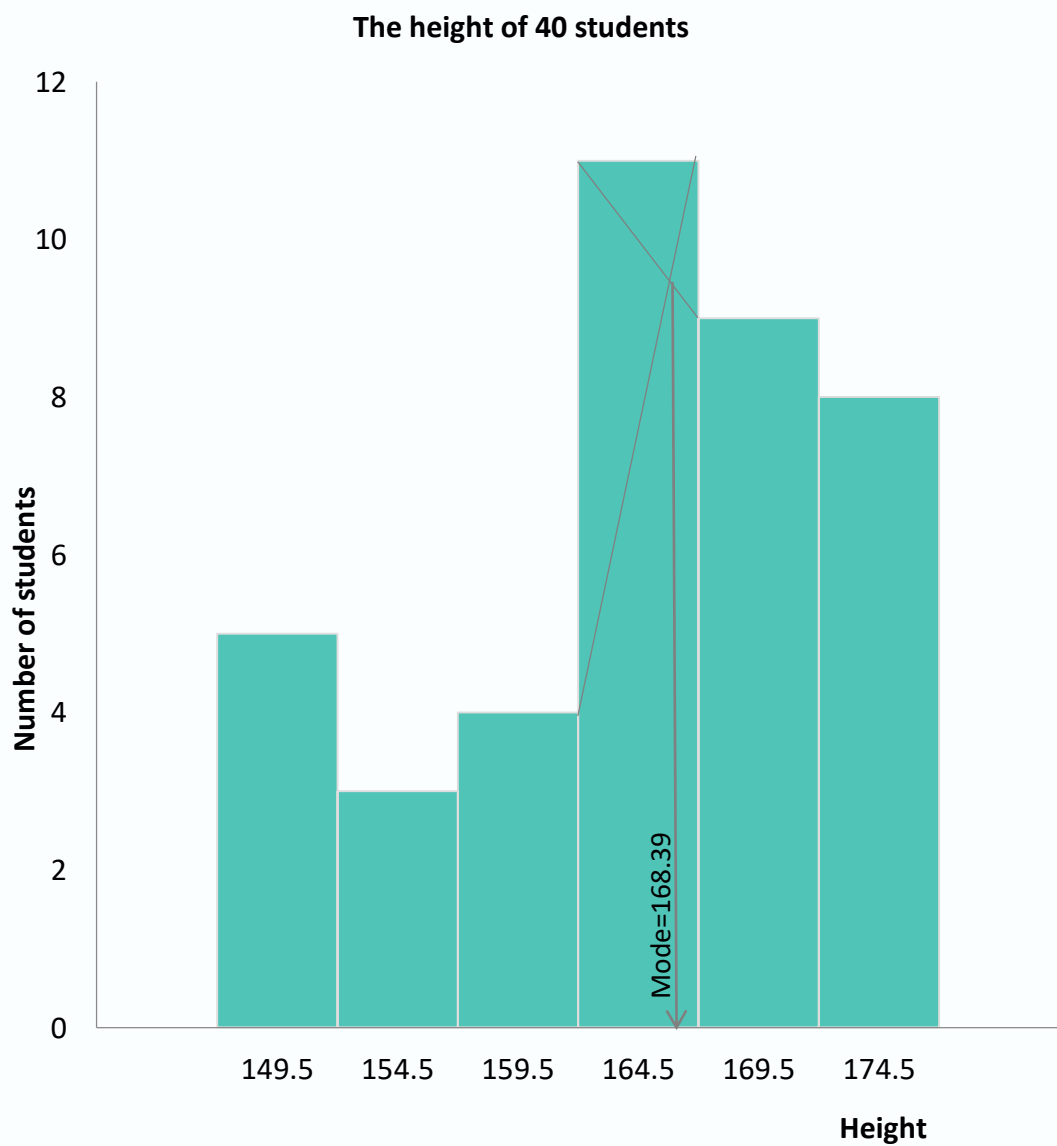


Figure 4 The height of 40 students

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

b. The median

Height (in cm)	Cumulative frequency	Upper boundaries
	0	149.5
150-154	5	154.5
155-159	8	159.5
160-164	12	164.5
165-169	23	169.5
170-174	32	174.5
175-179	40	179.5

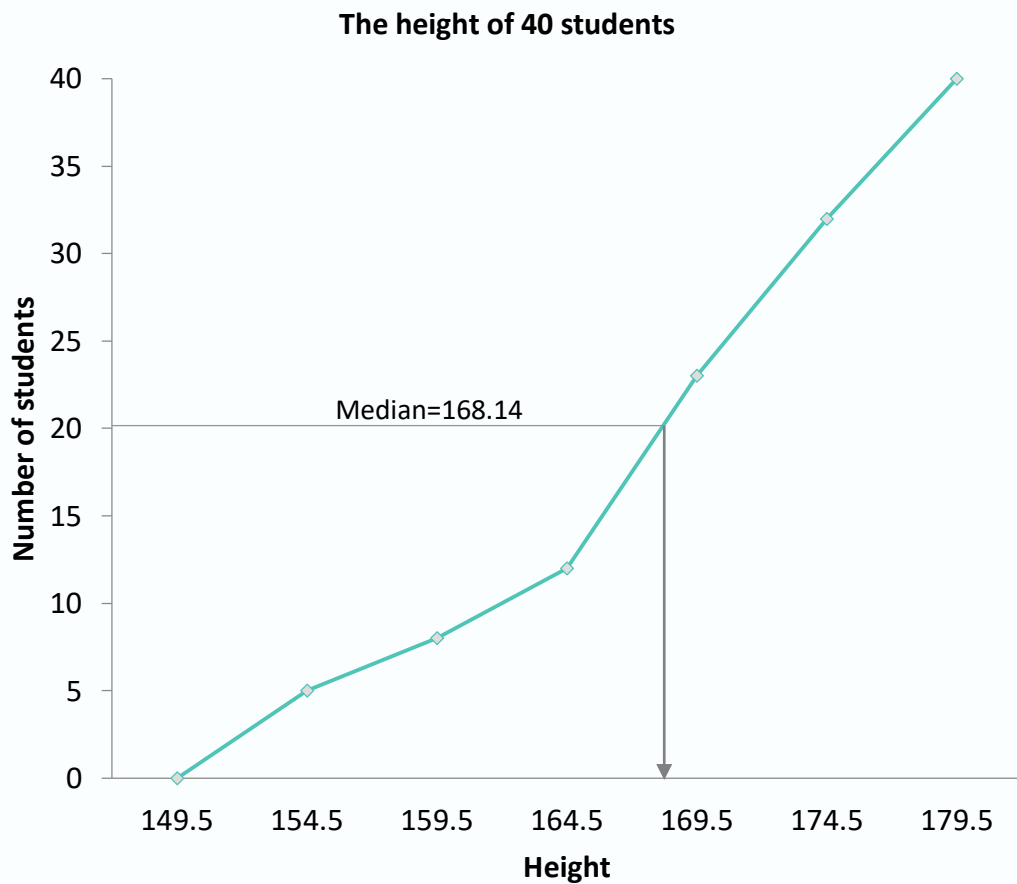


Figure 5 The height of 40 students

Exercise 1.2

1. Calculate the mean, median and mode for the following data.

a) 10 8 6 11 4 3 7 6 5 2

b) 2 5 5 8 3 1 7 5 9

2. The table below shows the number of pens bought by a students.

Number of students	3	7	5	4	1
Number of pens	1	2	3	4	5

Calculate mean, median and mode.

3. The table below shows the scores obtained by a group of students for a science quiz.

Number of students	5	6	1	3	10	m
Score	0	1	2	3	4	5

Given that the median is 2

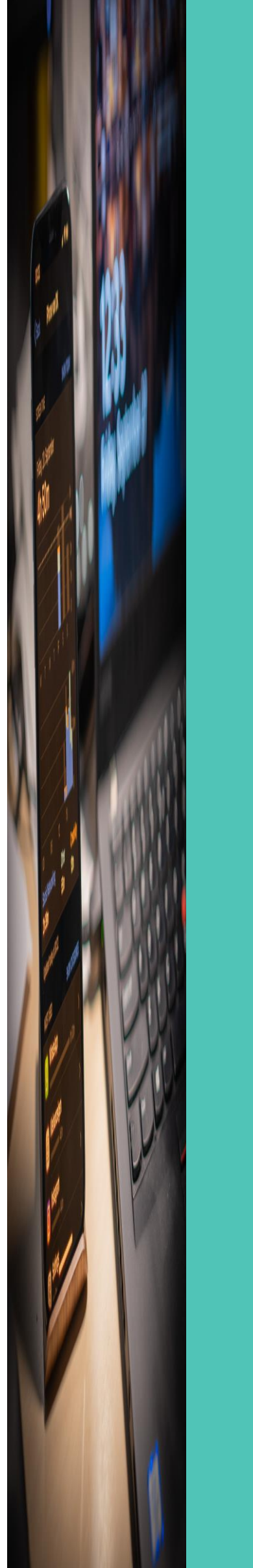
a) Find the value of m

b) Find the mean and mode

4. Table below shows the marks obtained by 50 trainees in a qualifying test to get accepted in an airlines agency.

Age (years)	Number of customer
50-59	13
60-69	17
70-79	9
80-89	6
90-99	5

Calculate the mean, median and mode.



COMPUTE OF CENTRAL TENDENCY AND DISPERSION

1.2.4 Calculate mean deviation, variance and standard deviation

- a. **Mean deviation** is the average of the absolute differences between each value in a set of values, and the average of all the values of that set .
- b. **Variance** is a measure of how much the values in a set of data vary from the mean of the set of data. A larger value of variance indicates a greater dispersion of the values from its mean.
- c. **Standard deviation** is also measure that tells us how much the values in a set of data disperse from the mean. Both ungrouped data and ungrouped data use the same formula because both use the value of variance.



MEASUREMENT	UNGROUPED DATA	GROUPED DATA
Mean deviation, E	$E = \frac{\sum x - \bar{x} }{n}$ <p>where x =data \bar{x} =mean n =total number of data</p>	$E = \frac{\sum x - \bar{x} f}{\sum f}$ <p>where x =data \bar{x} =mean $\sum f$ = total frequency</p>
Variance, s^2	$s^2 = \frac{(x - \bar{x})^2}{n}$ <p>where x =data \bar{x} =mean n =total number of data</p>	$s^2 = \frac{\sum (x - \bar{x})^2 f}{\sum f}$ <p>where x =data \bar{x} =mean $\sum f$ = total frequency</p>
Standard deviation, s	$s = \sqrt{\text{variance}}$	

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

Ungrouped Data

Example 12

Calculate the mean deviation, variance and standard deviation for the following ungrouped data:

Solution

a. Mean deviation, E

10	7	5	6	3	2	9
----	---	---	---	---	---	---

x	$ x - \bar{x} $
10	4
7	1
5	1
6	0
3	3
2	4
9	3
$\sum x = 42$	$\sum x - \bar{x} = 16$

$$\begin{aligned} \text{Mean, } \bar{x} &= \frac{\sum x}{n} \\ &= \frac{42}{7} \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{Mean deviation, } E &= \frac{\sum |x - \bar{x}|}{n} \\ &= \frac{16}{7} \\ &= 2.29 \end{aligned}$$

b. Variance, s^2

$ x - \bar{x} ^2$
16
1
1
0
9
16
9
$\sum x - \bar{x} ^2 = 52$

$$\begin{aligned} s^2 &= \frac{\sum (x - \bar{x})^2}{n} \\ &= \frac{52}{7} \\ &= 7.43 \end{aligned}$$



COMPUTE OF CENTRAL TENDENCY AND DISPERSION

c. Standard deviation, s

$$s = \sqrt{\text{variance}}$$

$$= \sqrt{7.43}$$

$$= 2.73$$

Grouped Data

Example 13

1. Calculate the mean deviation, variance and standard deviation for the following data:

Class	Frequency, f
16-21	15
22-27	16
28-33	5
34-39	5

Solution

a. Mean deviation, E

f	x	fx
15	18.5	277.5
16	24.5	392.0
5	30.5	152.5
5	36.5	182.5
$\Sigma f = 41$		$\Sigma = 1004.5$

$$\begin{aligned} \text{Mean, } \bar{x} &= \frac{\Sigma fx}{\Sigma f} \\ &= \frac{1004.5}{41} \\ &= 24.5 \end{aligned}$$

f	$ x - \bar{x} $	$f x - \bar{x} $
15	6	90
16	0	0
5	6	30
5	12	60
$\Sigma f = 41$		$\Sigma = 180$

$$\begin{aligned} E &= \frac{\Sigma |x - \bar{x}| f}{\Sigma f} \\ &= \frac{180}{41} \\ &= 4.39 \end{aligned}$$



COMPUTE OF CENTRAL TENDENCY AND DISPERSION

b. Variance, s^2

f	$ x - \bar{x} ^2$	$f x - \bar{x} ^2$
15	36	540
16	0	0
5	36	180
5	144	720
$\Sigma f = 41$		$\Sigma = 1440$

$$s^2 = \frac{\sum (x - \bar{x})^2 f}{\sum f}$$

$$= \frac{1440}{41}$$

$$= 35.12$$

c. Standard deviation, s

$$s = \sqrt{\text{variance}}$$

$$= \sqrt{35.12}$$

$$= 5.93$$



1.2.5 Calculate quartiles, deciles and percentiles by using formula and graph

1. First quartile (Q_1)

$$Q_1 = L_{Q_1} + \left[\frac{\frac{N}{4} - F}{f_{Q_1}} \right] C$$

L_{Q_1} = lower boundary of the class in which the first quartile lies

N = sum of frequency

F = cumulative frequency **before** the class in which the first quartile lies

f_{Q_1} = frequency of the class in which the first quartile lies

C = class size

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

Third quartile (Q_3)

$$Q_3 = L_{Q_3} + \left[\frac{\frac{3N}{4} - F}{f_{Q_3}} \right] C$$

L_{Q_3} = lower boundary of the class in which the third quartile lies

N = sum of frequency

F = cumulative frequency **before** the class in which the third quartile lies

f_{Q_3} = frequency of the class in which the third quartile lies

C = class size



Interquartile range

= Third quartile – First quartile

Decile

$$D_k = L_{Dk} + \left[\frac{\frac{k}{10}N - F}{f_{Dk}} \right] C$$

L_{Dk} = lower boundary of the class in which the decile lies

N = sum of frequency

F = cumulative frequency **before** the class in which the decile lies

f_{Dk} = frequency of the class in which the decile lies

C = class size

Percentile

$$P_k = L_{Pk} + \left[\frac{\frac{k}{100}N - F}{f_{Pk}} \right] C$$

L_{Pk} = lower boundary of the class in which the percentile lies

N = sum of frequency

F = cumulative frequency **before** the class in which the first quartile lies

f_{Pk} = frequency of the class in which the percentile lies

C = class size

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

Example 14

The table below shows the distribution of test scores obtained by 100 students in a mathematic class.

Scores obtained	No. of students
5-19	4
20-34	15
35-49	18
50-64	23
65-79	30
80-94	10
Total	100

Find,

- First quartile, Q_1
- Third quartile, Q_3
- Decile, D_5
- Percentile, P_{32}

Solution

Scores obtained	No. of students	Class boundaries	Cumulative frequency	Position of data
5-19	4	4.5-19.5	4	1-4
20-34	15	19.5-34.5	19	5-19
35-49	18	34.5-49.5	37	20-37
50-64	23	49.5-64.5	60	38-60
65-79	30	64.5-79.5	90	61-90
80-94	10	79.5-94.5	100	91-100
$\Sigma f = 100$				



COMPUTE OF CENTRAL TENDENCY AND DISPERSION



a. First quartile, Q_1

$$\begin{aligned}\text{Location of } Q_1 &= \frac{N}{4} \\ &= \frac{100}{4} \\ &= 25\end{aligned}$$

Q_1 class = 35-49

$$\begin{aligned}\text{First quartile, } Q_1 &= L_{Q_1} + \left[\frac{\frac{N}{4} - F}{f_{Q_1}} \right] C \\ &= 34.5 + \left[\frac{\frac{100}{4} - 19}{18} \right] 15 \\ &= 34.5 + 5 \\ &= 39.5\end{aligned}$$

b. Third quartile, Q_3

$$\begin{aligned}\text{Location of } Q_3 &= \frac{3N}{4} \\ &= \frac{3(100)}{4} \\ &= 75\end{aligned}$$

Q_3 class = 65-79

$$\begin{aligned}\text{Third quartile, } Q_3 &= L_{Q_3} + \left[\frac{\frac{3N}{4} - F}{f_{Q_3}} \right] C \\ &= 64.5 + \left[\frac{\frac{3(100)}{4} - 60}{30} \right] 15 \\ &= 64.5 + 7.5 \\ &= 72\end{aligned}$$

c. Decile, D_5

$$\begin{aligned}\text{Location of } D_5 &= \frac{k}{10} N \\ &= \frac{5}{10} (100) \\ &= 50\end{aligned}$$

D_5 class = 50-64

$$\begin{aligned}\text{Decile, } D_5 &= L_{Dk} + \left[\frac{\frac{k}{10} N - F}{f_{Dk}} \right] C \\ &= 49.5 + \left[\frac{\frac{5}{10}(100) - 37}{23} \right] 15 \\ &= 49.5 + 8.478 \\ &= 57.978\end{aligned}$$

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

d. Percentile, P_{32}

$$\begin{aligned}\text{Location of } P_{32} &= \frac{k}{100} N \\ &= \frac{32}{100} (100) \\ &= 32\end{aligned}$$

P_{32} class = 35-49

$$\begin{aligned}\text{Percentile, } P_{32} &= L_{Pk} + \left[\frac{\frac{k}{100}N - F}{f_{Pk}} \right] C \\ &= 34.5 + \left[\frac{\frac{32}{100}(100) - 19}{18} \right] 15 \\ &= 34.5 + 10.833 \\ &= 45.333\end{aligned}$$



Example 15

The table below shows the distribution of test scores obtained by 100 students in a mathematic class.

Scores obtained	No. of students
5-19	4
20-34	15
35-49	18
50-64	23
64-79	30
80-94	10
Total	100

Calculate by using the ogive graph,

- First quartile, Q_1
- Third quartile, Q_3
- Decile, D_5
- Percentile, P_{32}

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

Solution

Scores obtained	No. of students	Upper boundaries	Cumulative frequency
		4.5	0
5-19	4	19.5	4
20-34	15	34.5	19
35-49	18	49.5	37
50-64	23	64.5	60
65-79	30	79.5	90
80-94	10	94.5	100



Test Scores obtained by 100 Students in a Mathematic Class

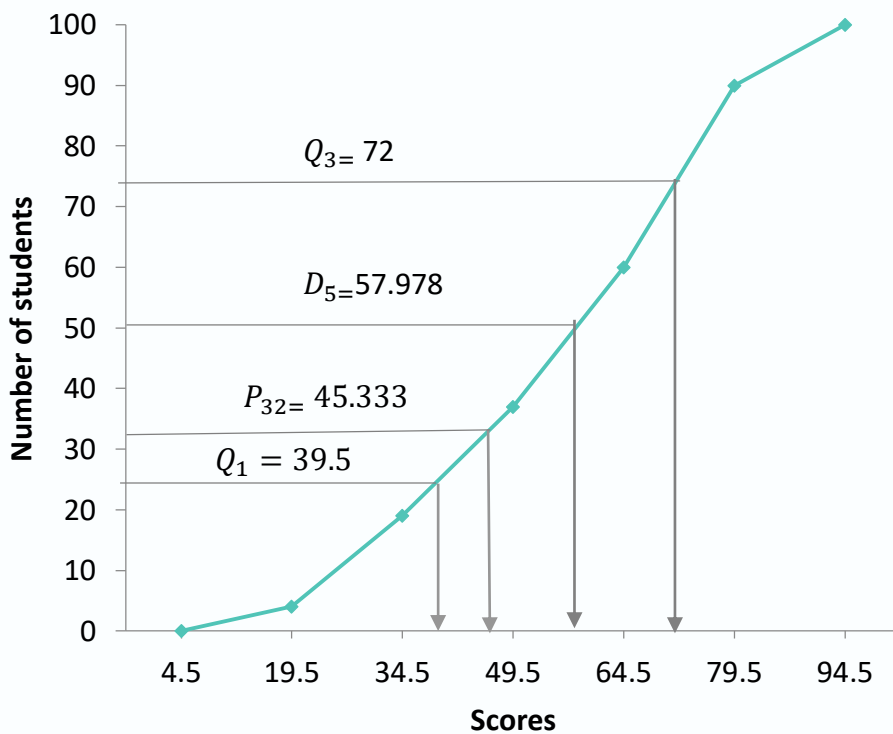


Figure 6 An ogive of test scores obtained by 100 students in mathematic class

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

a. First quartile, Q_1

$$Q_1 = \frac{1}{4} \times 100$$

$$= 25^{\text{th}}$$

$$Q_1 = 39.5$$

b. Third quartile, Q_3

$$Q_3 = \frac{3}{4} \times 100$$

$$= 75^{\text{th}}$$

$$Q_3 = 72$$

c. Decile, D_5

$$D_5 = \frac{5}{18} \times 100$$

$$= 50^{\text{th}}$$

$$D_5 = 57.978$$

d. Percentile, P_{32}

$$P_{32} = \frac{32}{100} \times 100$$

$$= 32^{\text{th}}$$

$$P_{32} = 45.333$$

Example 16

The table below shows the thickness distribution of 70 books in a box.

Thickness (cm)	No. of books
1.0-1.4	16
1.5-1.9	5
2.0-2.4	13
2.5-2.9	25
3.0-3.4	11



COMPUTE OF CENTRAL TENDENCY AND DISPERSION



Calculate

- First quartile, Q_1
- Third quartile, Q_3
- Decile, D_4
- Percentile, P_{50}

Solution

Thickness (cm)	No. of books	Class boundaries	Cumulative frequency	Position of data
1.0-1.4	16	0.95-1.45	16	1-16
1.5-1.9	5	1.45-1.95	21	17-21
2.0-2.4	13	1.95-2.45	34	22-34
2.5-2.9	25	2.45-2.95	59	35-59
3.0-3.4	11	2.95-3.45	70	60-70
	$\Sigma f = 70$			

- First quartile, Q_1

$$\begin{aligned} \text{Location of } Q_1 &= \frac{N}{4} \\ &= \frac{70}{4} \\ &= 17.5 \end{aligned}$$

Q_1 class = 1.5-1.9

$$\begin{aligned} \text{First quartile, } Q_1 &= L_{Q_1} + \left[\frac{\frac{N}{4} - F}{f_{Q_1}} \right] C \\ &= 1.45 + \left[\frac{\frac{70}{4} - 16}{5} \right] 0.5 \\ &= 1.45 + 0.15 \\ &= 1.6 \end{aligned}$$

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

b. Third quartile, Q_3

$$\begin{aligned}\text{Location of } Q_3 &= \frac{3N}{4} \\ &= \frac{3(70)}{4} \\ &= 52.5\end{aligned}$$

Q_3 class = 2.5-2.9

$$\begin{aligned}\text{Third quartile, } Q_3 &= L_{Q_3} + \left[\frac{\frac{3N}{4} - F}{f_{Q_3}} \right] C \\ &= 2.45 + \left[\frac{\frac{3(70)}{4} - 34}{25} \right] 0.5 \\ &= 2.45 + 0.37 \\ &= 2.82\end{aligned}$$

c. Decile, D_4

$$\begin{aligned}\text{Location of } D_4 &= \frac{k}{10} N \\ &= \frac{4}{10} (70) \\ &= 28\end{aligned}$$

D_4 class = 2.0-2.4

$$\begin{aligned}\text{Decile, } D_4 &= L_{Dk} + \left[\frac{\frac{k}{10} N - F}{f_{Dk}} \right] C \\ &= 1.95 + \left[\frac{\frac{4}{10}(70) - 21}{13} \right] 0.5 \\ &= 1.95 + 0.269 \\ &= 2.219\end{aligned}$$

d. Percentile, P_{50}

$$\begin{aligned}\text{Location of } P_{50} &= \frac{k}{100} N \\ &= \frac{50}{100} (70) \\ &= 35\end{aligned}$$

P_{50} class = 2.5-2.9

$$\begin{aligned}\text{Percentile, } P_{50} &= L_{Pk} + \left[\frac{\frac{k}{100} N - F}{f_{Pk}} \right] C \\ &= 2.45 + \left[\frac{\frac{50}{100}(70) - 34}{25} \right] 0.5 \\ &= 2.45 + 0.32 \\ &= 2.47\end{aligned}$$



COMPUTE OF CENTRAL TENDENCY AND DISPERSION

Example 17

The table below shows the thickness distribution of 70 books in a box.

Thickness (cm)	No. of books
1.0-1.4	16
1.5-1.9	5
2.0-2.4	13
2.5-2.9	25
3.0-3.4	11

Calculate by using the ogive graph

- First quartile, Q_1
- Third quartile, Q_3
- Decile, D_4
- Percentile, P_{50}



Solution

Thickness (cm)	No. of books	Upper boundaries	Cumulative frequency
		0.95	0
1.0-1.4	16	1.45	16
1.5-1.9	5	1.95	21
2.0-2.4	13	2.45	34
2.5-2.9	25	2.95	59
3.0-3.4	11	3.45	70

- First quartile, Q_1

$$Q_1 = \frac{1}{4} \times 70$$

$$= 17.5^{\text{th}}$$

$$Q_1 = 1.6$$

- Third quartile, Q_3

$$Q_3 = \frac{3}{4} \times 70$$

$$= 52.5^{\text{th}}$$

$$Q_3 = 2.82$$

COMPUTE OF CENTRAL TENDENCY AND DISPERSION

c. Decile, D_4

$$D_4 = \frac{4}{10} \times 70$$

$$= 28^{\text{th}}$$

$$D_4 = 2.219$$

d. Percentile, P_{50}

$$P_{50} = \frac{50}{100} \times 70$$

$$= 35^{\text{th}}$$

$$P_{50} = 2.47$$

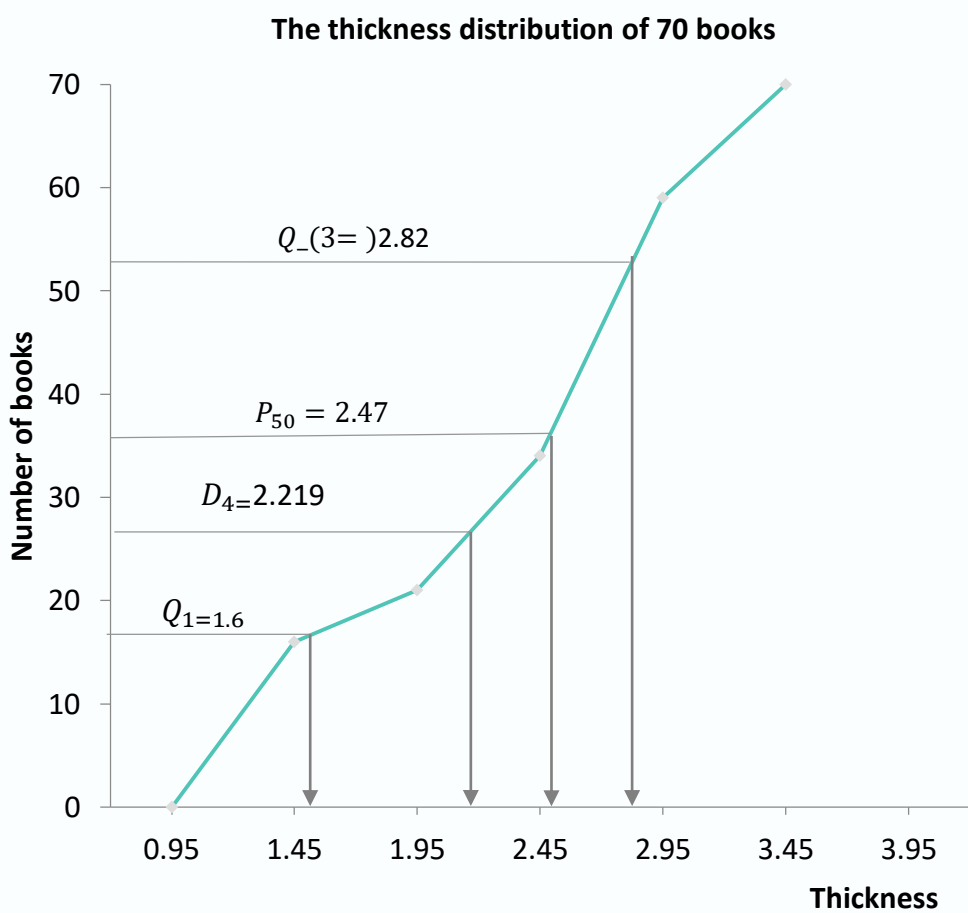


Figure 7 An ogive of the thickness distribution of 70 books in a box

Exercise 1.3

1. Calculate the mean, variance and standard deviation of the following set of data.

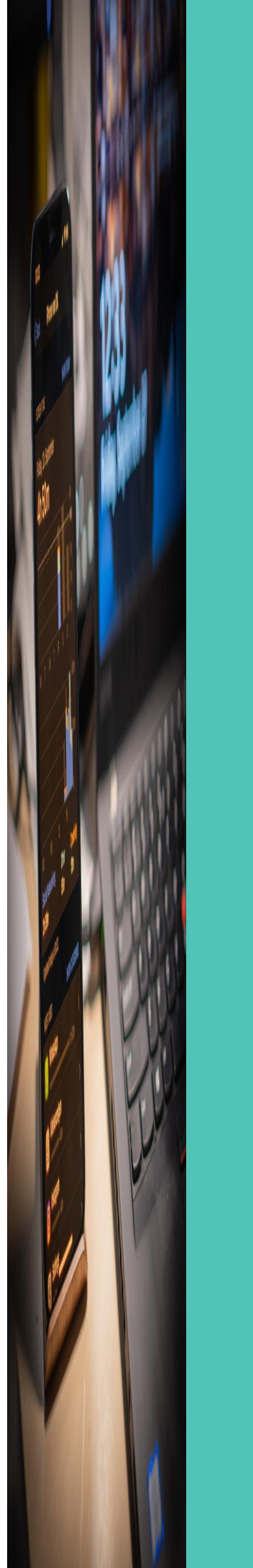
48 53 55 58 60 51 57 47 59 52

2. Table shows the distribution frequency of time period taken by 100 students solving the mathematical problem .

Time (minute)	Frequency
6-10	8
11-15	17
16-20	20
21-25	19
26-30	18
31-35	11
36-40	7

Calculate

- a. Mean
- b. Mean deviation
- c. Standard deviation



COMPUTE PROBABILITY



1.3 Compute Probability

Probability is the likelihood or chance that a particular event will occur. Probability could refer to the chance of picking a black card from a deck of cards, the chance that an individual prefers one product over another, or the chance that a new consumer product on the market will be successful. In each of these examples, probability is a proportion or fraction whose values range between 0 and 1, inclusively. Note that an event that has no chance of occurring (the impossible event) has a probability of 0, while an event that is sure to occur (the certain event) has a probability of 1.

A Sample Space

Definition: The possible outcomes of a random experiment are called the **basic outcomes**, and the set of all basic outcomes is called the **sample space**. The symbol S will be used to denote the sample space.

Example 18

A dice is rolled. The basic outcomes are the numbers 1,2,3,4,5 or 6. Thus, the sample space is

Solution

$$S = \{1, 2, 3, 4, 5, 6\}$$

Generally, interest is not in the basic outcomes themselves but in some subset of all the outcomes in the sample space.

An Event

Definition: An event is a subset of the sample space. An event is a set of outcomes that satisfy certain specific conditions.

Example 19

Ten card labelled $A, B, C, D, E, F, G, H, I, J$ are placed inside a box. Let E be the event a vowel card is obtained. Write the event, E .

Solution

Event, $E = \{A, E, I\}$

COMPUTE PROBABILITY

Probability of an Event

If S is a sample space of an experiment, then the probability of event A , written as $P(A)$, is defined as

$$P(A) = \frac{\text{number of elements in } A}{\text{number of elements in sample space } S}$$

A is a subset of S , i.e. $A \subset S$ and

$$0 \leq P(A) \leq 1$$

Proof

$A \subset S$, then $0 \leq n(A) \leq n(S)$

Divide by $n(S)$,

$$\frac{0}{n(S)} \leq \frac{n(A)}{n(S)} \leq \frac{n(S)}{n(S)}$$

$$0 \leq P(A) \leq 1$$

If $P(A) = 1$ this means that the event is an absolute certainty.

If $P(A) = 0$ this means that the event is an absolute impossibility.

Example 20

If one ball is taken from a bag containing only red balls:

$$P(\text{ball is red}) = 1 \text{ and } P(\text{ball is blue}) = 0$$

1.3.1 Define the following types of event

Expectation

The **expectation**, E , of an event happening is defined in general terms as the product of the probability p of an event happening and the number of attempts made, n ; i.e., $E = pn$.

Example 21

Find the expectation of obtaining a number 2 with 3 throws of a dice.

Solution

The probability, p , of obtaining a number 2 for 1 throw of a dice is $\frac{1}{6}$.

Three attempts are made, $n = 3$

The expectation, E , is pn .

$$\text{So, } E = \frac{1}{6} \times 3 = \frac{1}{2}$$



COMPUTE PROBABILITY



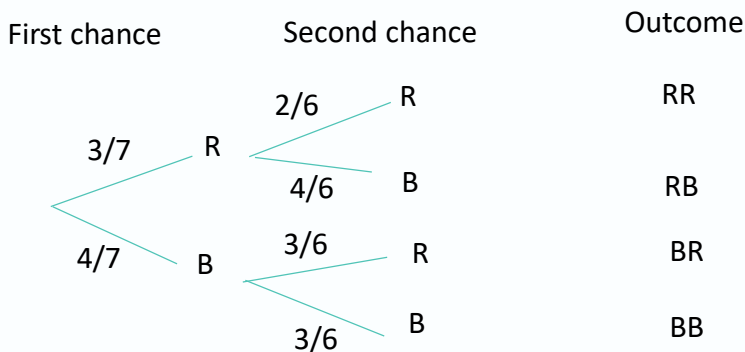
Dependent Event

A **dependent event** is one in which the probability of an event happening **affects** the probability of another event happening. **Without replacement**, the events are dependent.

Example 20

A box contains 3 red and 4 blue chips. Two chips are drawn at random without replacement. Calculate the probability of getting 2 red chips?

Solution



Therefore

$$\frac{3}{7} \times \frac{2}{6} = \frac{1}{7}$$

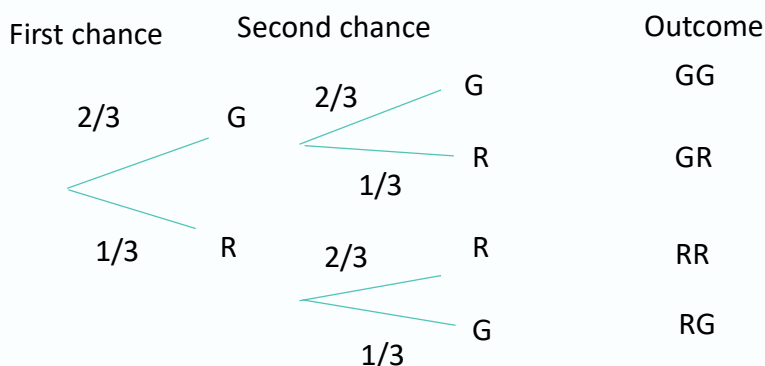
Independent Event

An Independent event is one in which the probability of an event happening **does not affect** the probability of another event happening. **With replacement**, the events are independent.

Example 21

A basket contains 4 green balls and 2 red balls. Two balls are drawn at random with replacement. Calculate the probability of getting both balls are red?

Solution



Therefore

$$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

COMPUTE PROBABILITY

Example 22

A fair coin is tossed and a dice is rolled. Determine the probability of getting head of the coin and getting a number 2 on the dice.

Solution

Let H= event of getting heads

A= event of getting a number 2

$$P(H) = \frac{1}{2}$$

$$P(A) = \frac{1}{6}$$

$$P(H \text{ and } A) = P(H) \times P(A)$$

$$= \frac{1}{2} \times \frac{1}{6}$$

$$= \frac{1}{12}$$

Conditional Probability

Conditional probability is the probability of an event, given some other event has already occurred.

The probability of A given B is equal to the probability of A and B divided by probability.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

The probability of B given A is equal to the probability of A and B is divided by the probability of A:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

where:

$P(A \text{ and } B)$ = joint probability of A and B

$P(A)$ = marginal probability of A

$P(B)$ = marginal probability of B

Example 23

Two events A and B have probabilities $P(A) = 0.6$, $P(B) = 0.4$ and $P(A \cap B) = 0.1$, $P(B \cap A) = 0.3$. Calculate

a) $P(A|B)$ b) $P(B|A)$

Solution

a) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.4} = 0.25$

b) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.3}{0.6} = 0.5$



COMPUTE PROBABILITY

Example 24

A school has 30 teachers; 40% of them are males and the rest are females. Eight of the male teachers are married while 13 of the females are married. A teacher is randomly selected from the school. If it is known that the teacher selected is single, what is the probability that the teacher is a male?

Solution

We can construct table below by the given information.

Status	Male	Female	Total
Single	4	5	9
Married	8	13	21
Total	12	18	30

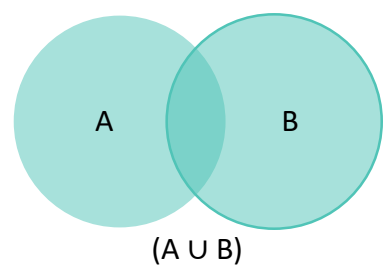
$$\begin{aligned}
 P(\text{male} \mid \text{single}) &= \frac{P(\text{male} \cap \text{single})}{P(\text{single})} \\
 &= \frac{4/30}{9/30} \\
 &= \frac{4}{9}
 \end{aligned}$$

The Probability of Two Events

The probability of an event A or B happening can be determined by the union of sets A and B ($A \cup B$) and the formula is:

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)}$$

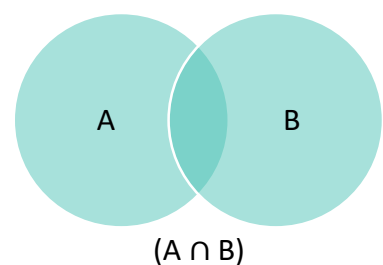
The combined event can be represented by the Venn Diagram



The probability of events A and B happening at the same time can be determined by the intersection of sets A and B ($A \cap B$) and the formula is:

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

The combined event can be represented by the Venn Diagram



COMPUTE PROBABILITY

1.3.2 Use laws of probability

Addition Law of Probability

The addition law of probability is recognized by the word 'or' joining probabilities.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A \cup B)$ is the event that A occurs and B occurs or both events A and B occur.

$P(A \cap B)$ is the event that both A and B occur together.

$$P(A) \neq 0, \text{ and } P(B) \neq 0$$



Example 25

The probabilities of events R and S are such that $P(R) = \frac{6}{16}$, $P(S) = \frac{1}{4}$ and $P(A \cap B) = \frac{3}{16}$

Find $P(R \cup S)$

Solution

$$P(R \cup S) = P(R) + P(S) - P(R \cap S)$$

$$= \frac{6}{16} + \frac{1}{4} - \frac{3}{16}$$

$$= \frac{10}{16} - \frac{3}{16}$$

$$= \frac{7}{16}$$

$$= 0.4375$$

Example 26

Ten cards numbered from 41 to 50 are placed in box. A card is picked at random from the box. Find the probability of picking a number that is odd or divisible by 3.

Solution

$$S = \{41, 42, 43, 44, 45, 46, 47, 48, 49, 50\}$$

$$n(S) = 10$$

Let A = event of picking an odd number

$$= \{41, 43, 45, 47, 49\}$$

$$n(A) = 5$$

B = event of picking a number that is divisible by 3

$$= \{45\}$$

$$n(B) = 1$$

COMPUTE PROBABILITY

$$A \cap B = \{45\}$$

$$n(A \cap B) = 1$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

$$= \frac{1}{10}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{10} + \frac{1}{10} - \frac{1}{10}$$

$$= \frac{5}{10}$$

$$= \frac{1}{2}$$



If two events, A and B are **mutually exclusive**, that is both events cannot occur at the same time or simultaneously. In other words, mutually exclusive events are called disjoint events. This means that if A occurs, B cannot occur or if B occurs, A cannot occur. Thus, for two mutually exclusive events A and B can be written as

$$P(A \cup B) = P(A) + P(B) ; A \cap B = \emptyset$$



Example 27

MUTUALLY EXCLUSIVE

Tossing a coin : Heads and Tails

Cards : King and Aces

Turning left and turning right (can't do both at the same time)

Example 28

A dice is rolled. What is the probability of a dice showing a 3 or 6.

$$S = \{1, 2, 3, 4, 5, 6\}$$

Solution

$$n(S) = 6$$

Let A = {event of showing a 3}

$$n(A) = 1$$

COMPUTE PROBABILITY

$B = \{\text{event of showing a 6}\}$

$$n(B) = 1$$

$$P(A) = \frac{1}{6} \quad \text{and} \quad P(B) = \frac{1}{6}$$

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{1}{3}$$

Example 29

A basket contains 4 green balls, 6 blue balls and 8 red balls. A ball is selected random from the basket.

Find the probability of selecting a green or red ball

Solution

$S = \{4 \text{ green balls, } 6 \text{ blue balls, } 8 \text{ red balls}\}$

$$n(S) = 18$$

Let G = event green ball is selected

B = event blue ball is selected

R = event red ball is selected

$$P(G \cup R) = P(G) + P(R)$$

$$= \frac{n(G)}{n(S)} + \frac{n(R)}{n(S)}$$

$$= \frac{4}{18} + \frac{8}{18}$$

$$= \frac{12}{18}$$

$$= \frac{2}{3}$$

Multiplication Law of Probability

The multiplication law of probability is recognized by the word '**and**'.

If events **A and B are independent**, then

$$P(A \cap B) = P(A) \times P(B)$$

where \cap means 'and'

If events **A and B are dependent**, the multiplication rule is modified as

$$P(A \cap B) = P(A) \times P(B|A)$$

In case when we have dependent events we have to be very careful in determining the probability of the second event after the occurrence of first event. A tree diagram can be constructed to show all the possible outcomes of an experiment. Each branch shows the possible outcomes of an event.



COMPUTE PROBABILITY

Example 30

Four yellow cards and two green cards are placed in the box. A card is selected from the box.

With replacement, what the chances of selecting two green cards?

Solution

Let G = event that a green card is selected.

$$P(G) = \frac{2}{6}$$

Let Y = event that a yellow card is selected.

$$P(Y) = \frac{4}{6}$$

Therefore,

$$P(G \cap G) = P(G) \times P(G)$$

$$= \frac{2}{6} \times \frac{2}{6}$$

$$= \frac{4}{36}$$

$$= \frac{1}{9}$$

Example 31

A basket has 5 pink balloons and 2 white balloons. A balloon is selected from the basket without replacement.

Find the probability of selecting both pink balloons.

Let A = event that the first balloon is pink.

Let B = event that the second balloons is pink.

Solution

$$P(A \cap B) = P(A) \times P(B|A)$$

$$= \frac{5}{7} \times \frac{4}{6}$$

$$= \frac{20}{42}$$

$$= \frac{11}{21}$$



COMPUTE PROBABILITY

1.3.3 Solve problems on probability

Example 32

Bag A contains four balls numbered 3,4,5 and 6 respectively. Bag B contains three balls numbered 4,5 and 7 respectively. A ball is drawn at random from each bag. Calculate the probability that:

- The number of both ball are different.
- The number of both ball are same.
- The sum of the numbers on the two balls does not exceed 10.

Solution

$$a. S = \{(3,4), (3,5), (3,7), (4,4), (4,5), (4,7), (5,4), (5,5), (5,7), (6,4), (6,5), (6,7)\}$$

$$n(S) = 12$$

$$A = \{ \text{The number of both ball are different} \}$$

$$A = \{(3,4), (3,5), (3,7), (4,5), (4,7), (5,4), (5,7), (6,4), (6,5), (6,7)\}$$

$$n(A) = 10$$

$$P(A) = \frac{10}{12}$$

$$= \frac{5}{6}$$

OR using Tree diagram

$$\begin{aligned} P(\text{The number of both ball are different}) &= \left(\frac{1}{4} \times \frac{1}{3}\right) \times 10 \\ &= \frac{10}{12} \\ &= \frac{5}{6} \end{aligned}$$

$$b. P(\text{The number of both ball are same}) = 1 - P(A)$$

$$= 1 - \frac{5}{6}$$

$$= \frac{1}{6}$$

$$c. B = \{ \text{The sum of the numbers on the two balls does not exceed 10} \}$$

$$B = \{(3+4), (3+5), (3+7), (4+4), (4+5), (5+4), (5+5), (6+4)\}$$

$$n(B) = 8$$

$$P(B) = \frac{8}{12}$$

$$= \frac{2}{3}$$



COMPUTE PROBABILITY

c. $B = \{ \text{The sum of the numbers on the two balls does not exceed } 10 \}$

$$B = \{(3 + 4), (3 + 5), (3 + 7), (4 + 4), (4 + 5), (5 + 4), (5 + 5), (6 + 4)\}$$

$$n(B) = 8$$

$$P(B) = \frac{8}{12} \\ = \frac{2}{3}$$



P (The sum of the numbers on the two balls does not exceed 10)

$$= \left(\frac{1}{4} \times \frac{1}{3}\right) \times 8 \\ = \frac{8}{12} \\ = \frac{2}{3}$$

Example 33

A survey of middle asked: what is your favourite sport when school holiday. The results are summarized below:

Grade	Football	Futsal	Badminton	Total
3rd	68	41	46	155
4th	84	56	70	210
5th	59	74	47	180
Total	211	171	163	545

From this study you have to randomly select:

- What is probability a selecting a student whose favourite sport is futsal?
- What is probability a selecting a 3rd grade student?
- What is probability a selecting a student whose favourite sport is futsal and 3rd grade student?

Solution

a. $P(\text{futsal}) = \frac{171}{545}$

b. $P(3^{\text{rd}}) = \frac{155}{545}$

a. $P(\text{futsal} \cap 3^{\text{rd}}) = \frac{41}{545}$

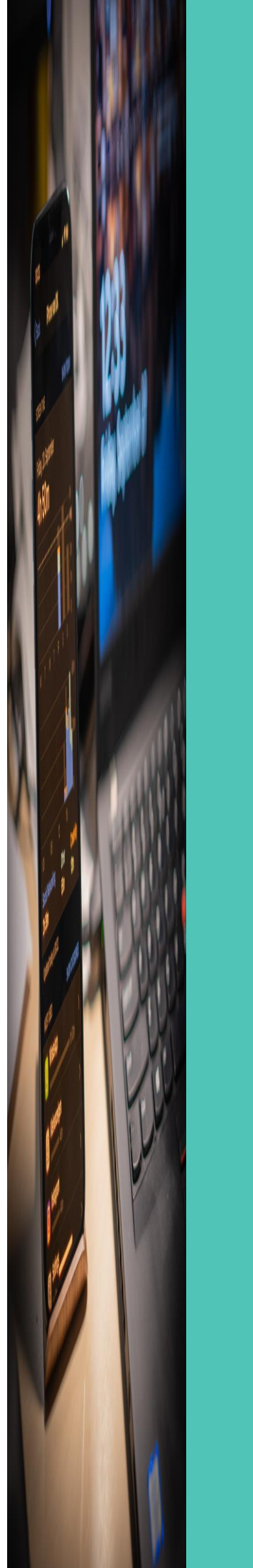
Exercise 1.4

1. Describe the sample space of each of the following experiments.
 - a) A fair dice is rolled
 - b) A fair coin is tossed
 - c) A bag contains a black ball, a white ball and a yellow ball. A ball is drawn randomly from the bag.

2. A letter is randomly selected from 'EPAL'
 - a) Describe the sample space of this experiment
 - b) Determine the number of possible outcomes of event that selected letter is a consonant and a vowel.

3. There are 25 green marbles and 20 yellow marbles in a box. r green marbles are then added to the box so that the probability of obtaining a yellow marble from the box is $\frac{4}{5}$. Calculate the value of r .

4. In Selangor, 75 % of all students have a computer. In addition, 37% of all students have a computer and a television. What is the probability that a student has a television given that he or she has a computer?



REVIEW QUESTIONS

1. Calculate the mean deviation, variance and standard deviation for the following set of data.

21 24 17 11 8 7
9 5 6 21 18 19

2. Find the value of x if the mean of 4, 5, 6, 7, 11 and x is 8.
3. a) Find the mean, mode and median for the data set below.

50 30 10 30 40 10 10 20 50 10

- b) The table shows the quiz marks obtained by 50 students of DAD3S2. Find the mean deviation, variance and standard deviation for these data:

Score	10-19	20-29	30-39	40-49
Frequency	10	20	5	15

4. Table shows the typing speed for 120 secretarial students.

Time (Minute)	Number of Students
1-10	30
11-20	15
21-30	43
31-40	20
41-50	10
51-60	2

Based on the above table, calculate

- Mean
- Median
- Variance
- Standard deviation

5. A pencil box contains 9 red pencils, 6 blue pencils and m black pencils. If the probability of picking a blue pencil is $\frac{3}{10}$, find the value of m .

6. A bag contains 3 green marbles, 4 yellow marbles and 5 purple marbles. Find the probability that the marble drawn is

- a. Purple
- b. Yellow or green
- c. Blue

7. a) i) A container contains of 10 red plates, 15 blue plates, 8 yellow plates and 7 white plates. A plate is picked randomly from the container. What is the probability of picking a blue plate?

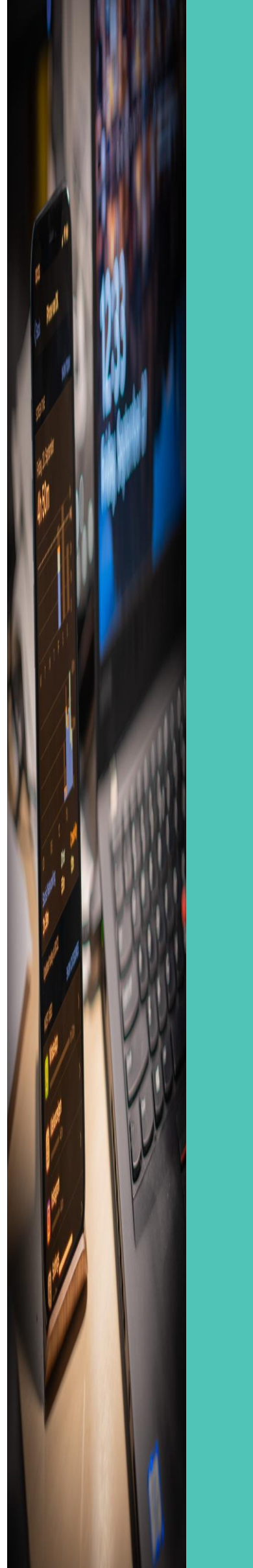
ii) A roulette wheel is divided into 10 equal sectors labelled as E, L, E, C, T, R, I, C, A, L. The wheel is spin twice. Find the probability that the wheel stopped on the letter A on the first spin and the letter A on the second spin.

iii) The following table shows the number of books in a box. Two books are selected from the box. Without replacing the books, what is the probability of getting 1 Bahasa Malaysia book and 1 English book?

Subject	Number of books
Mathematics	4
Bahasa Malaysia	8
English	5

b) i. The probabilities of Danial and Damia to be chosen as members of a committee are $\frac{3}{5}$ and $\frac{7}{9}$ respectively. Find the probability that neither of them is chosen as a member of the committee.

ii. The probabilities of Ahmad and Aina to be chosen as members of a committee are $\frac{2}{3}$ and $\frac{5}{8}$ respectively. Find the probability that only one of them is chosen as a member of the committee.



8. There are 25 red balls and 20 blue balls in a basket. r blue balls are then added to the basket so that the probability of obtaining a blue balls from the basket is $\frac{4}{5}$. Calculate the value of r .

9.a) A used car company started a business with 60 Proton and Honda cars. 25 of the cars are Proton. If two cars is sold and without replacement, determine the probability that:

- i. Both cars are Honda
- ii. Both cars are Proton
- iii. Both cars are different

b) There are 40 students from Sukma College: 25 students learn archery and 23 students learn swimming. Calculate the probability that

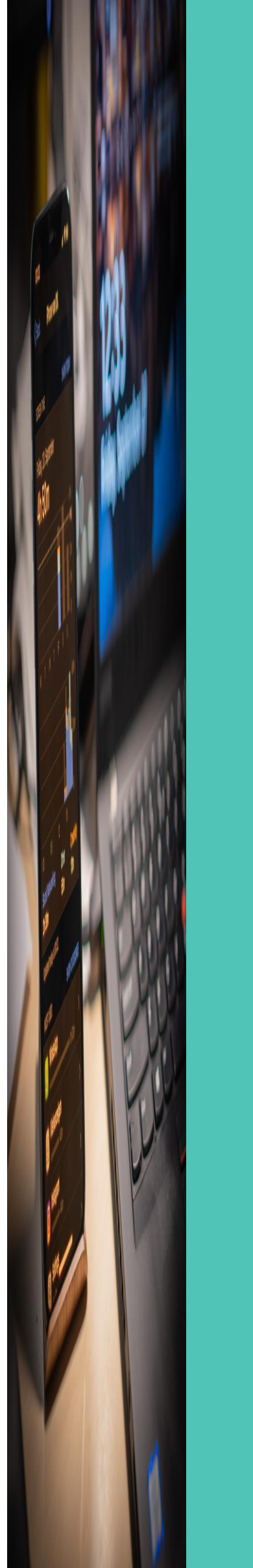
- i. A student learning archery or swimming.
- ii. A student learning archery and swimming
- iii. A student archery only
- iv. A student learning swimming only

10. a) i. A container contains of 20 red glasses, 32 blue glasses, 17 yellow glasses and 11 white glasses. A glass is picked randomly from container. What is the probability of picking a blue glass?

ii. A roulette wheel is divided into 10 equal sectors labelled as P, O, L, I, T, E, K, N, I and K. The wheel is spun twice. Find the probability that the wheel stopped on the letter I on the first spin and the letter K on the second spin.

iii. Table below shows the number of donuts in a box. Two donuts are selected from the box. Without replacing the donuts, what is the probability of getting 1 chocolate donut and 1 pink donut?

Donuts	Number of donuts
Original Donuts	9
Chocolate Donuts	12
Pink Donuts	10



b) A survey was done at a secondary school. The students was asked “ What is your favorite sport?” The results are summarized in table below:

Sport House	FootBall	Takraw	Softball	Total
Hang Tuah	74	54	52	180
Hang Jebat	96	45	60	201
Hang Lekir	98	60	55	213

By using these students as a sample space, a student from this study is randomly selected. What is the probability that :

- i. Selecting a student whose favorite sport is takraw?
- ii. Selecting a student from Hang Tuah sport house?
- iii. The student selected is from Hang Jebat sport house and not prefer softball?
- iv. The student selected is from Hang Lekir sport house and student prefers football or softball?

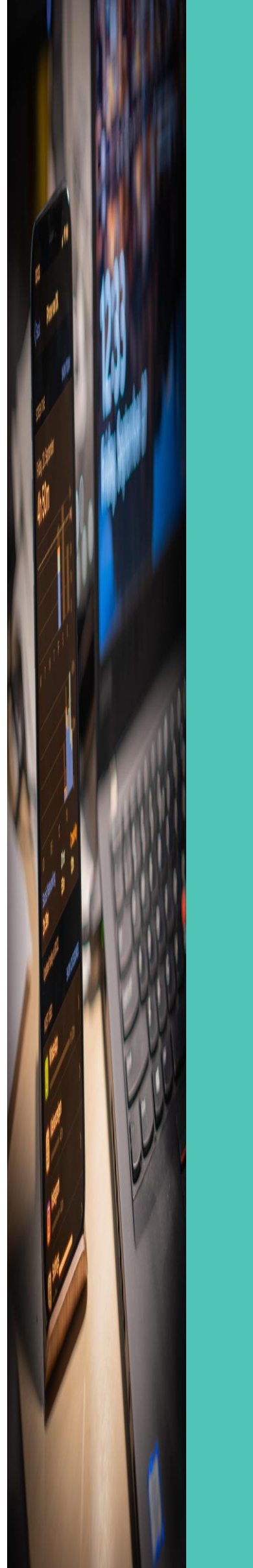
11. Table shows the number of boys and girls in three groups A, B and C. A kid is chosen randomly from each groups. Determine the probability of choosing three kids that :

Group	Number of boys	Number of girls
A	3	2
B	4	4
C	4	5

- a) All of them are girls.
- b) All of them are boys
- c) Consists of a girl and two boys

12. Two coins are tossed simultaneously. Calculate the probability of obtaining:

- a) Two heads
- b) Two tails
- c) No tails
- d) A head and a tail



ANSWERS

CHAPTER 1

Exercise 1.1

- 1 (a) discrete data (b) continuous data (c) discrete data
(d) continuous data (e) discrete data

2

MARK	TALLY	FREQUENCY
163-167		5
168-172		7
173-177		6
178-182		5
183-187		5
188-192		2

3

MARK	TALLY	FREQUENCY
4		2
5		2
6		4
7		5
8		4
9		2
10	/	1

ANSWERS

Exercise 1.2

- 1 mean=6.2 median=6 and mode=6
- 2 mean=2.65, median=2, mode=2.5
- 3 $m=8$, mean=0, mode=2.256
- 4 mean= 69.1, median= 66.56, mode=62.83

Exercise 1.3

- 1 mean, $\bar{x}=54$, variance, $s^2 =15.11$, standard deviation, $s=3.89$
- 2 (a) Mean=22.15 (b) Mean deviation=7.035 (c) Standard deviation=8.398

Exercise 1.4

- 1 (a) $s = \{ 1, 2, 3, 4, 5, 6 \}$
(b) $s = \{ H, T \}$
(c) $s = \{ \text{Black, white, yellow} \}$
- 2 (a) $S = \{ E, P, A, L \}$
(b) $P(\text{consonant}) = \frac{1}{2}$ $P(\text{vowel}) = \frac{1}{2}$
- 3 $r = 80$
- 4 0.493

ANSWERS

Review Questions

- 1 (a) mean deviation = 6.17 , variance= 42.64, standard deviation=6.53
- 2 $X = 15$
- 3 Mean = 29.5 , Mean Deviation= 10 , Variance = 125 , Standard Deviation= 11.18
- 4 (i) Mean= 23.08 (ii) Median=23.99
(iii) Variance= 171.66 (iv) Standard deviation=13.10
- 5 $m = 5$
- 6 (a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) 0
- 7 (a) (i) $\frac{15}{40}$ (ii) $\frac{2}{100}$ (iii) $\frac{80}{272}$
(b) (i) $\frac{4}{45}$ (ii) $\frac{11}{24}$
- 8 $r = 80$

ANSWERS

Review Questions

- 9** (i) 0.362 (ii) 0.1695 (iii) $\frac{175}{354}$
(b) (i) 1 (ii) $\frac{1}{5}$
(iii) $\frac{17}{40}$ (iv) $\frac{15}{40}$
- 10** (a) (i) $\frac{2}{5}$ (ii) $\frac{1}{25}$ (iii) $\frac{8}{31}$
(b) (i) $\frac{53}{198}$ (ii) $\frac{10}{33}$
(iii) $\frac{47}{67}$ (iv) $\frac{51}{71}$
- 11** (a) $\frac{1}{9}$ (b) $\frac{2}{15}$ (c) $\frac{7}{18}$
- 12** (a) $\frac{1}{4}$ (b) $\frac{1}{4}$
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

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e ISBN 978-967-2240-21-1

